

# **Surface Waves**

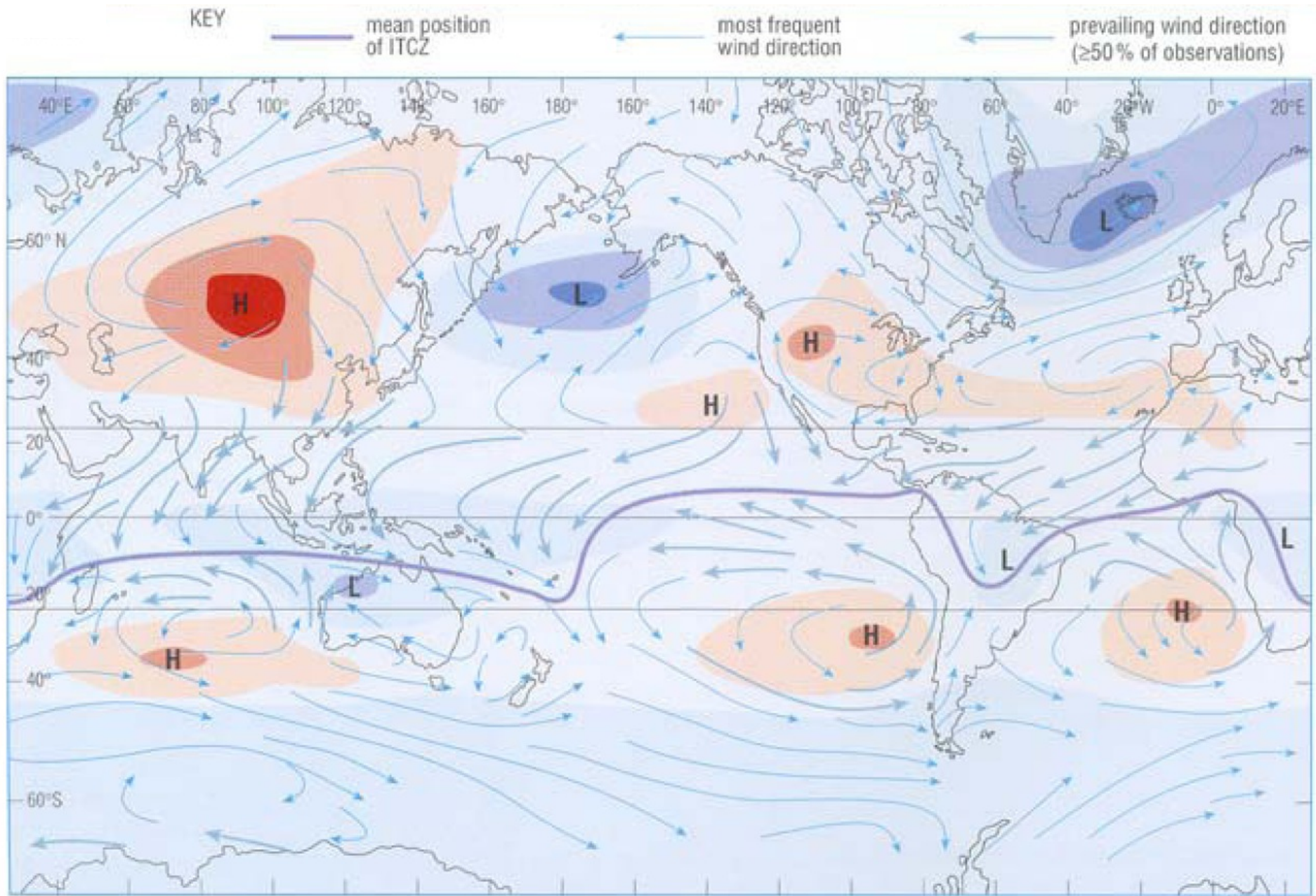
NOAA Tech Refresh

20 Jan 2012

Kipp Shearman, OSU

# Outline

- Surface winds
  - Wind stress
  - Beaufort scale
  - Buoy measurements
- Surface Gravity Waves
  - Wave characteristics
  - Deep/Shallow water waves
  - Generation
  - Modeling

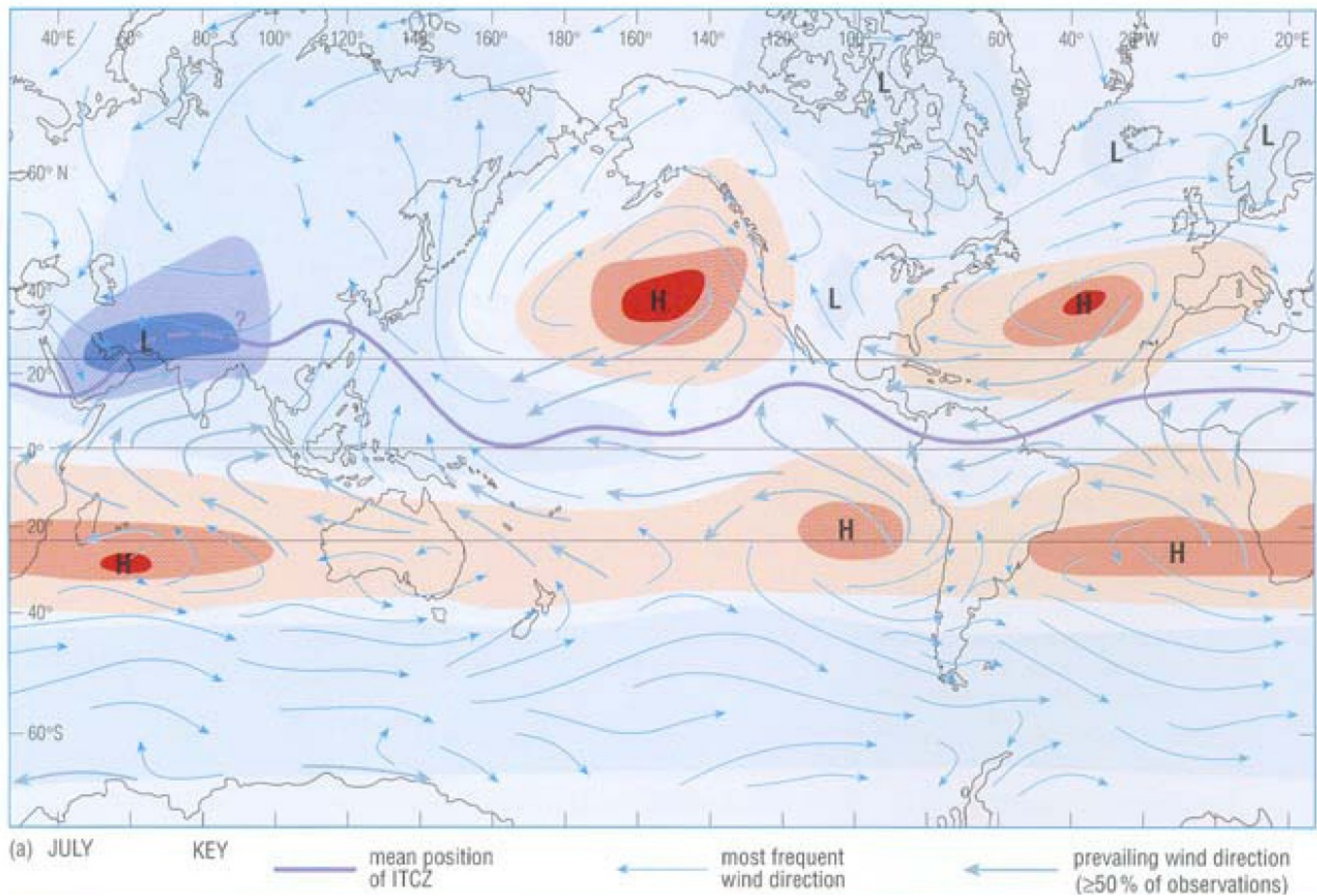


(b) JANUARY

January mean atmospheric circulation and surface winds

Fig 2.3 Ocean Circulation



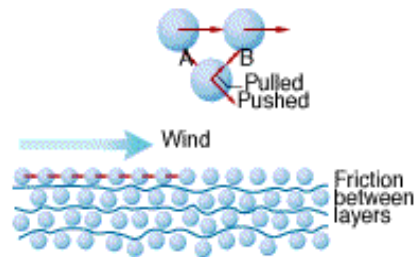


July mean atmospheric circulation and surface winds

Fig 2.3 Ocean Circulation

# Wind Forcing at the Ocean Surface

- Wind-forcing can generate *currents and waves*, as wind transfers some of its momentum into the ocean
- Wind acts via friction at the surface: wind stress  $\tau$



Stresses have units of  $\text{N/m}^2$ , (force/area), like pressure. Stresses are forces parallel to a surface, pressure is force perpendicular to the surface.

- Force/Area depends on the square of the wind speed  $u$ , and it points in the same direction as the wind:

$$\tau \propto u^2$$

$$\vec{\tau} = \rho_a C_D |\vec{u}| \vec{u}$$

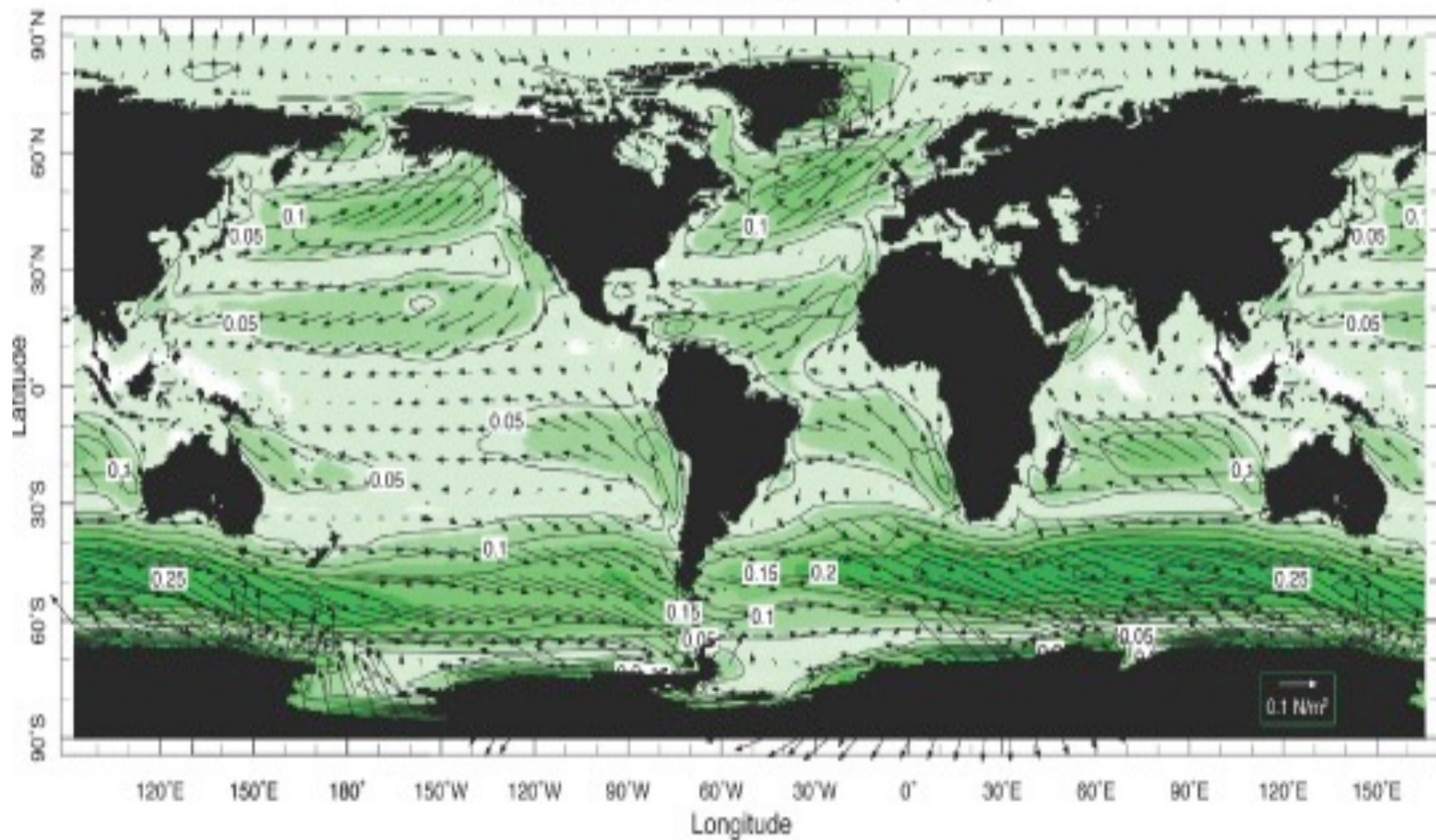
$$C_D = \text{drag coefficient} \approx 1.4 \times 10^{-3}$$

$$\rho_a = \text{density of air} \approx 1.3 \text{ kg} / \text{m}^3$$

Example: 20kt wind  $\approx 10 \text{ m/s} \rightarrow 0.18 \text{ N/m}^2 = 1.8 \text{ dyne/cm}^2$



## Surface Wind Stress ( $\text{N/m}^2$ )







0, <1 knot



1, 1-3 knots



2, 4-6 knots



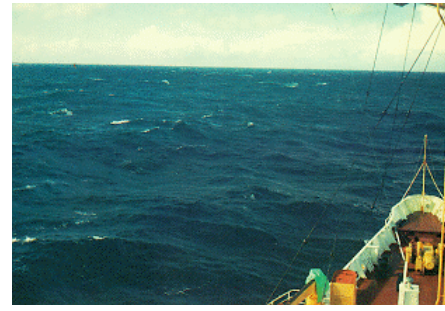
3, 7-10 knots



4, 11-17 knots



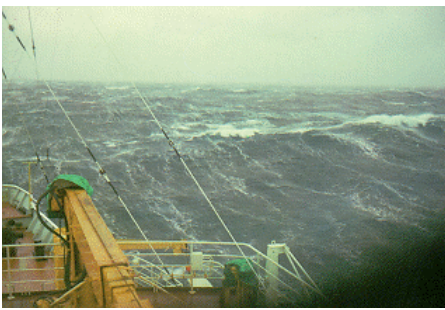
5, 17-21 knots



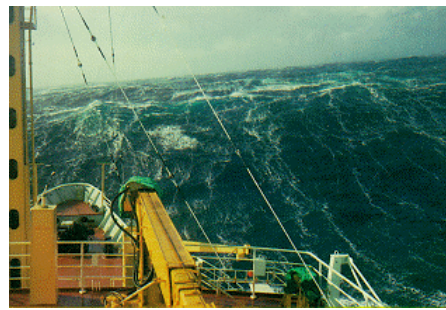
6, 22-27 knots



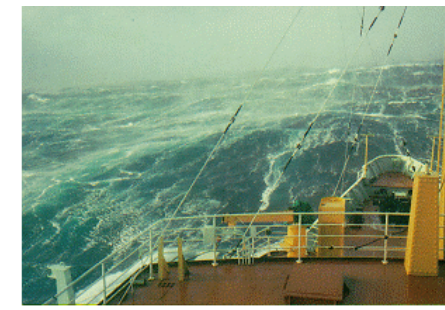
7, 28-33 knots



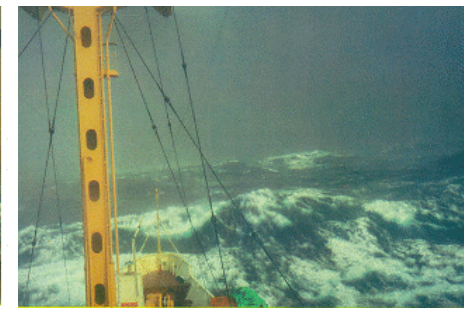
8, 33-40 knots



9, 41-47 knots



10, 48-55 knots



11, 56-63 knots

Beaufort scale, after <http://www.geology.wmich.edu/Kominz/windwater.html>

Beaufort scale still provides climatological data

Look at buoy winds off oregon coast.



<http://www.wrh.noaa.gov/pqr/>



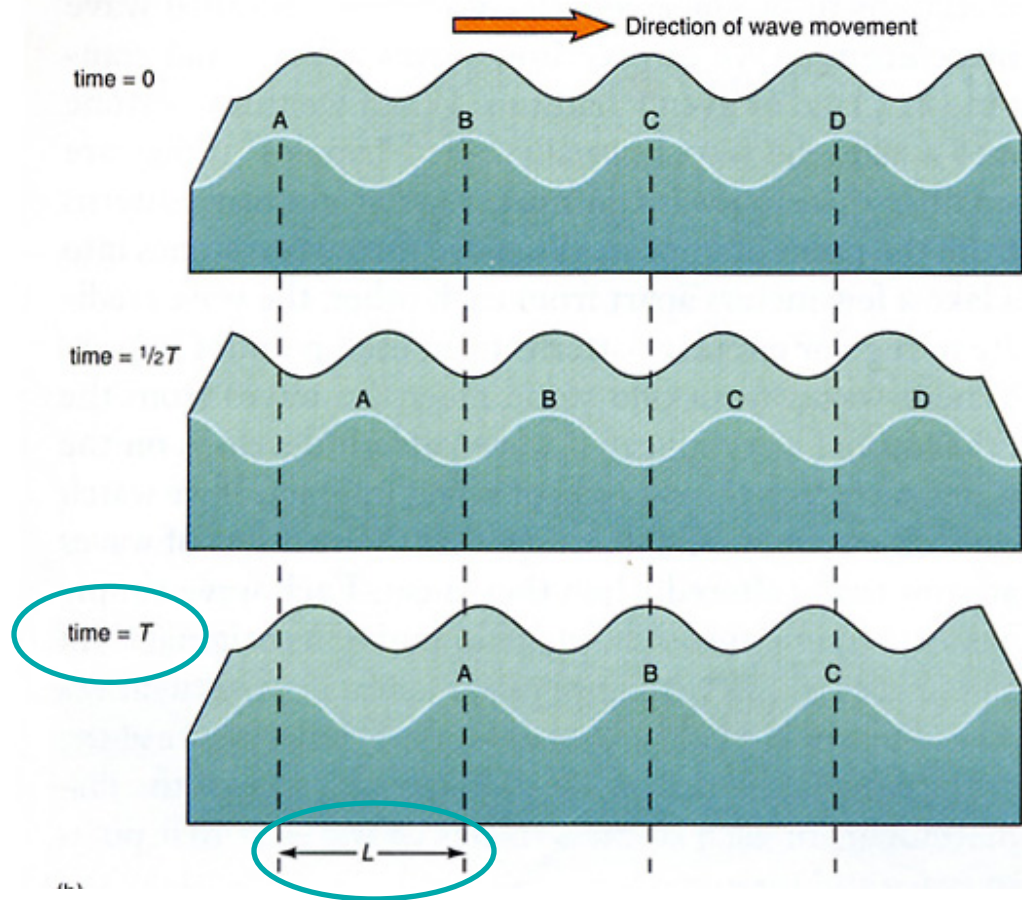
# Outline

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  - Beaufort scale
  - Buoy measurements
- **Surface Gravity Waves**
  - **Wave characteristics**
  - **Deep/Shallow water waves**
  - **Generation**
  - **Modeling**

# Wave Propagation

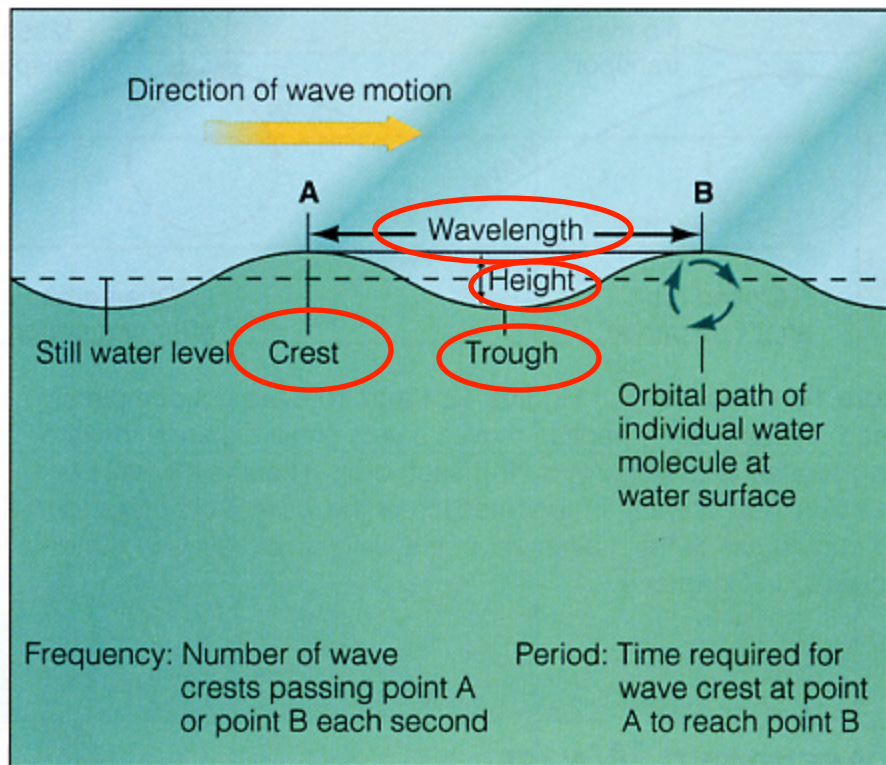
- Wave Period  $T$ :  
time between arrival of successive wave crests at a given place
- Wave Length  $L$ :  
distance between successive wave crests
- Wave (phase) speed  $c$ :  
$$c_p = L / T$$

**By definition, the definition of  $c_p$  is valid for all waves.**



Segar

# Structure/Terminology for a surface wave on the ocean



- Crest: highest point on the wave
- Trough: lowest point on the wave
- Height **H**: vertical distance between crest and trough
- (Amplitude **A**: half of Height)
- Wavelength **L**: distance between adjacent wave crests

**Figure 10.2** The anatomy of a progressive wave.

Garrison



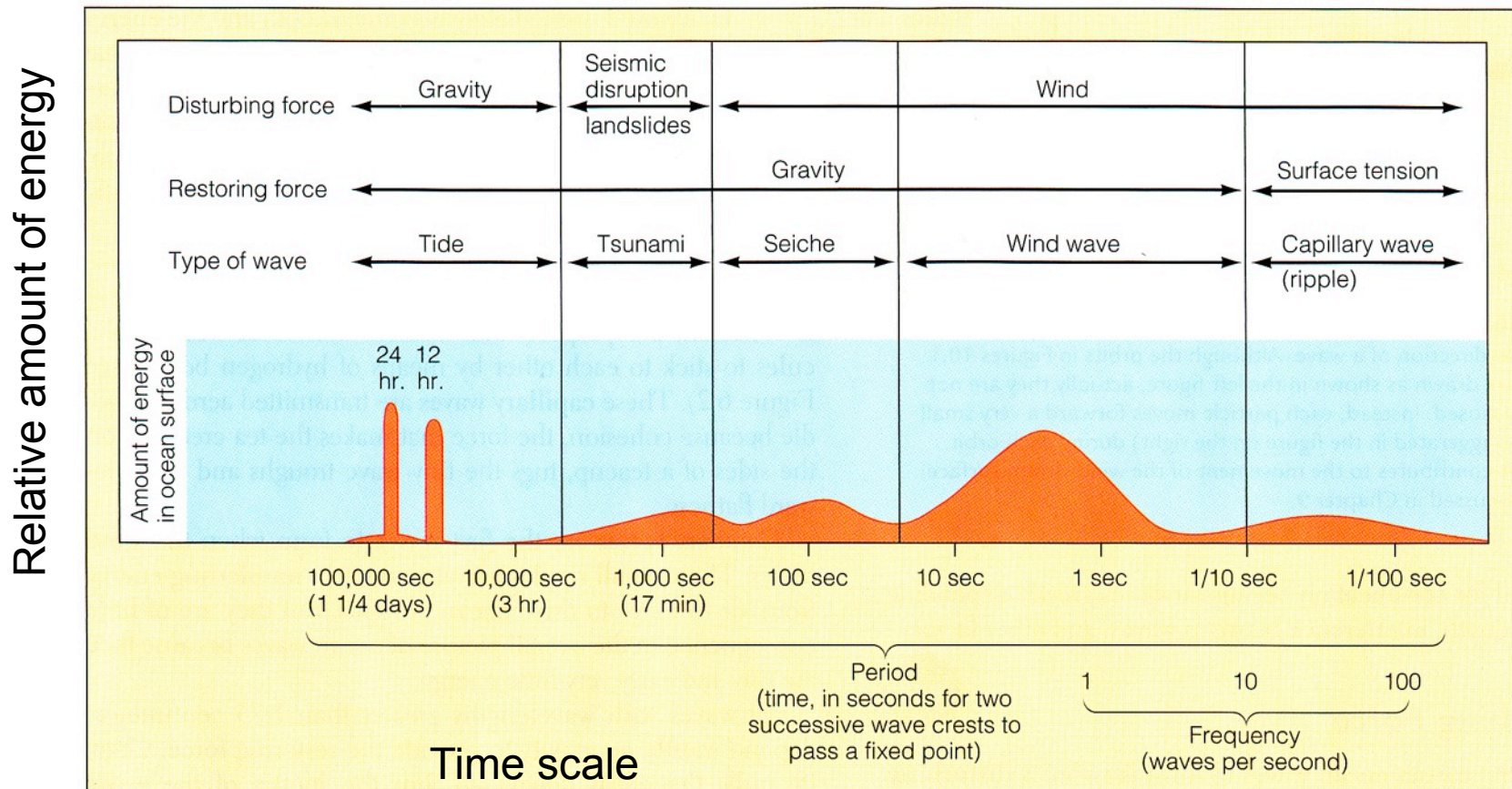
# Other definitions

- L often written  $\lambda$
- Wave number,  $k = 2\pi/L$
- Radial frequency,  $\sigma = 2\pi/T$  (also  $\omega$ )
- Frequency,  $f = 1/T$

# Ocean Waves

- **Disturbance:** sea surface is displaced
  - winds
  - vertical displacement
  - produce pressure disturbance
- **Restoring Force:**
  - gravity
  - surface tension
  - Earth's rotation
- **Propagation:**
  - pressure gradients due to different water heights
- **Dissipation**
  - bottom friction
  - wave breaking

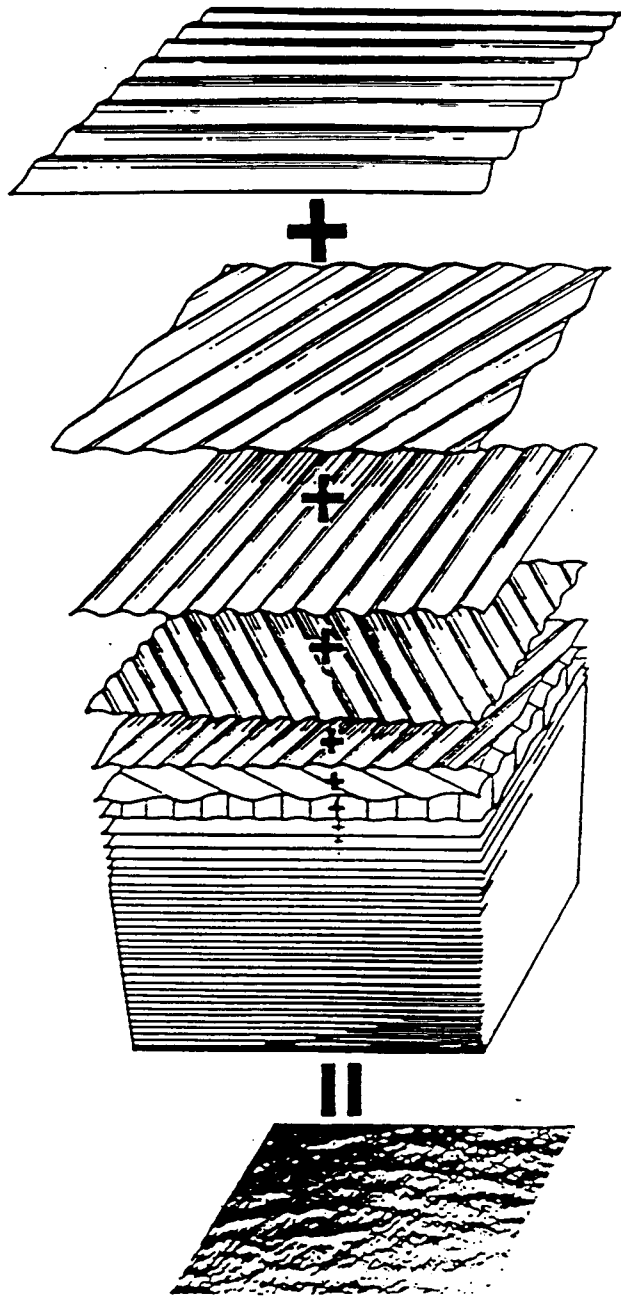
The ocean is a medium for a wide variety of waves with different sources and restoring forces.



**FIGURE 10.5** Wave energy in the ocean as a function of the wave period. As the graph shows, most wave energy is typically concentrated in wind waves. However, large tsunamis, rare events in the ocean, can transmit more energy than all wind waves for a brief time. Tides are waves—their energy is concentrated at periods of 12 and 24 hours.

Note, what this figure refers to as wind waves includes locally generated wind waves and ocean swell.





Under linear theory, the sea surface can be represented by the sum of many simple sinusoidal wave trains of different periods and amplitudes moving in different directions.

From Pierson et al. (1955)

## General dispersion relation for linear gravity waves

$$C_p = \sigma/k = L/T = \sqrt{\frac{gL}{2\pi} \tanh\left(\frac{2\pi d}{L}\right)}$$

- Derived from linearized conservation of momentum and volume with surface and bottom boundary conditions
- Good for any wavelength/depth combination
- $C_p$  is (phase) speed
- $g$  is gravitational acceleration (9.8 m/s<sup>2</sup>)
- $L$  is wavelength
- $\pi$  is 3.14159...
- Depth,  $d$
- Two limiting cases to consider

## Deepwater (shortwave) dispersion relation

$$C_p = \sigma/k = L/T = \sqrt{\frac{gL}{2\pi}}$$

- Good for  $2\pi d/L \gg 1$  ( $kd \gg 1$ ),  $d > L/2$
- Longer waves travel faster (dispersive)
- Know any one of  $c_p$ ,  $T$ ,  $L$  can get the others

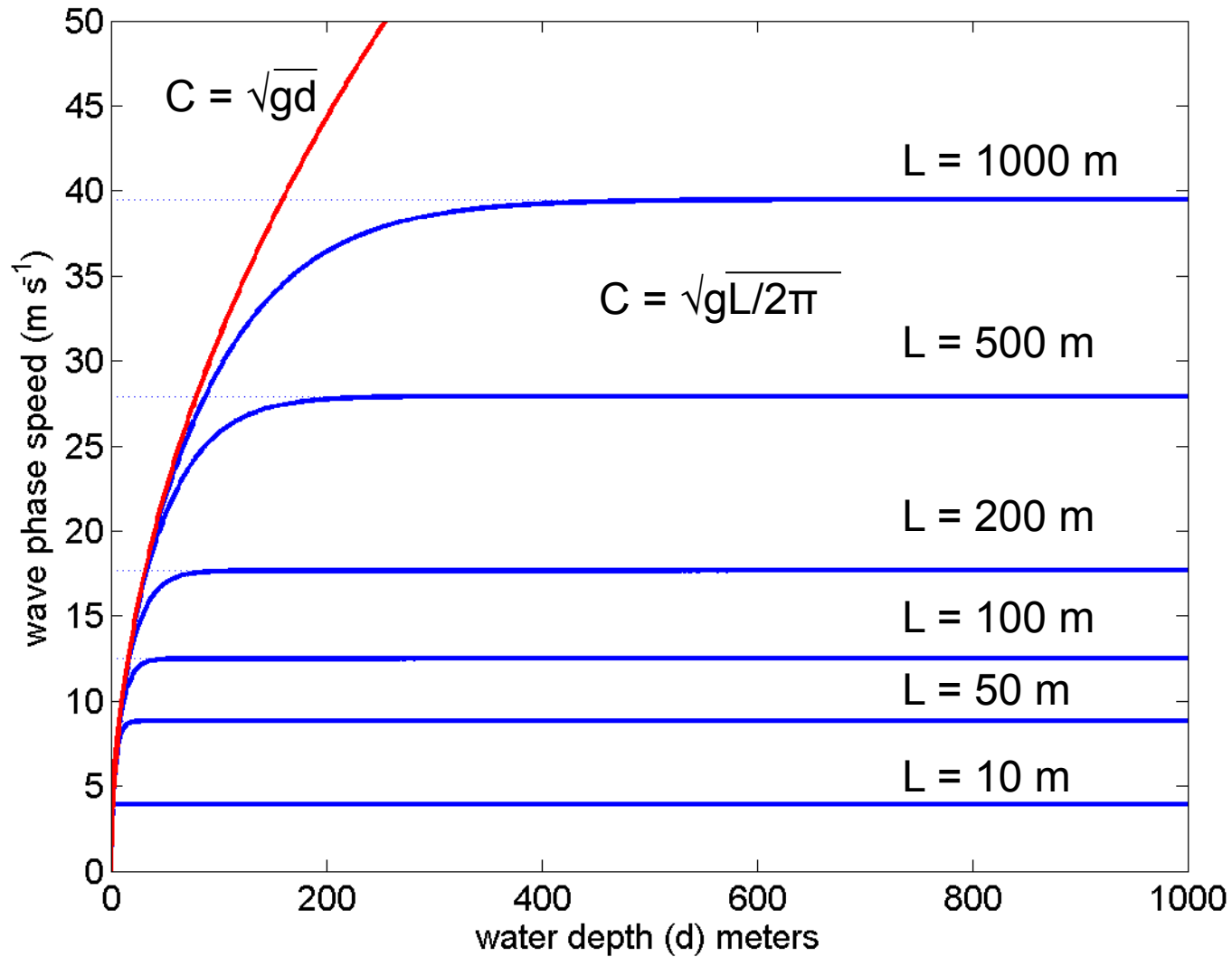


## Shallow water (long wave) dispersion relation

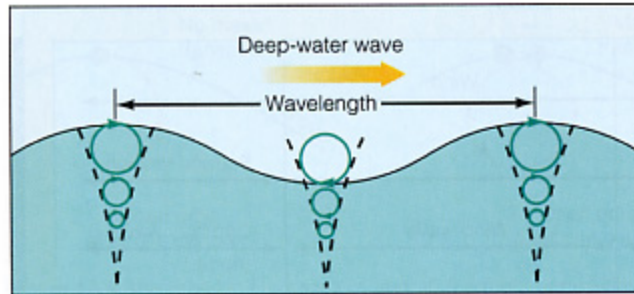
$$C_p = \sigma/k = L/T = \sqrt{gd}$$

- Good for  $2\pi d/L \ll 1$  ( $kd \ll 1$ ) or  $d < L/20$
- $c_p$  and  $d$  alone do not determine  $T$  and  $L$
- $c_p$  does not depend on wavelength so all wavelengths travel at same speed (non-dispersive)

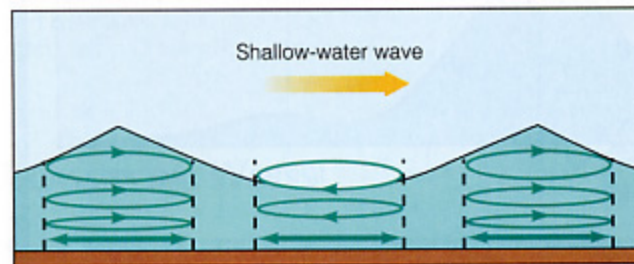
General (blue) and shallow water (red) dispersion relation plotted for several different wavelengths,  $L$



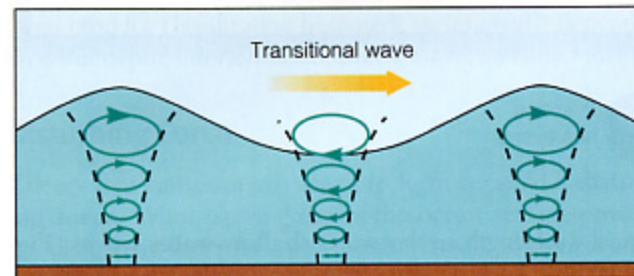
# What is the motion of the water under waves?



a Depth  $\geq \frac{1}{2}$  wavelength



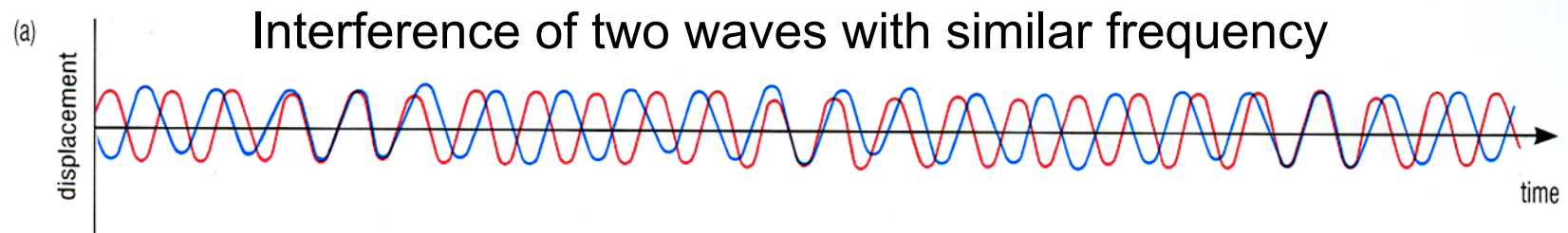
b Depth  $\leq \frac{1}{20}$  wavelength



c  $\frac{1}{20}$  wavelength  $\leq$  depth  $\leq \frac{1}{2}$  wavelength

**Figure 10.6** Progressive waves: (a) a deep-water wave; (b) a shallow-water wave; (c) a transitional wave. These diagrams are not to scale.

- Deep water wave:
  - nearly circular paths, shrink exponentially with depth
  - crests move with wave direction
  - wave motion  $\frac{1}{2}$  at  $L/9$ , essentially zero at depth of  $L/2$
- Shallow water wave:
  - more along-wave motion than vertical
  - more flattened toward the bottom
- Transitional water depths:
  - intermediate



“seventh wave” phenomenon

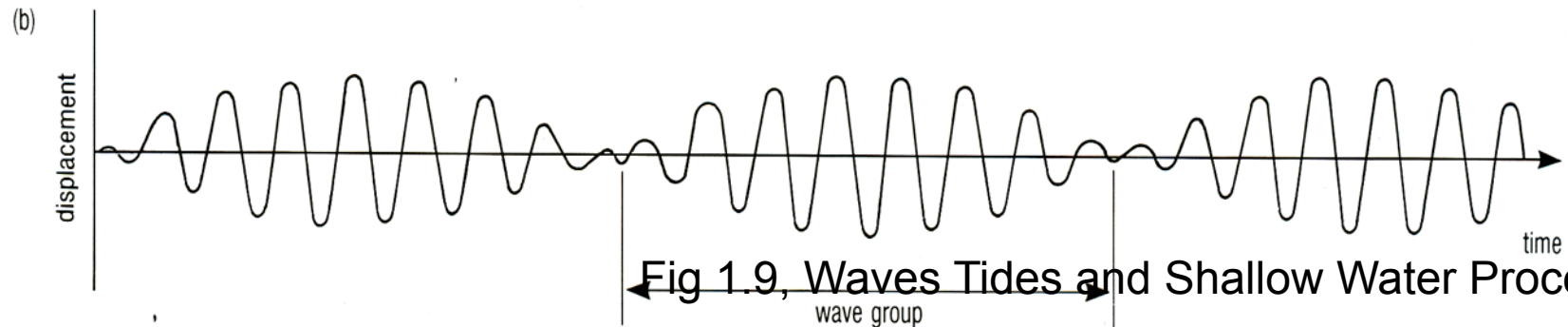


Fig 1.9, Waves Tides and Shallow Water Processes

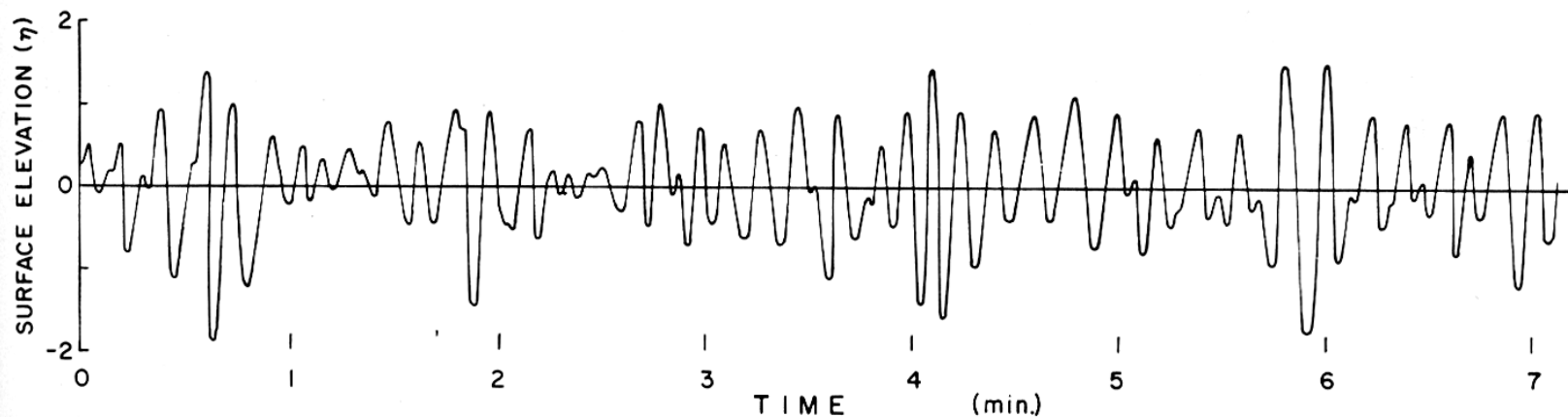


FIG. 12.10 Character of real wave train (profile) to compare with ideal sine waves of Figs. 12.2 and 12.3. (Note that the vertical scale is much exaggerated as before.)

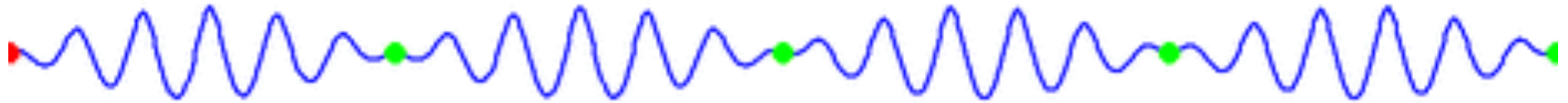
Pond and Pickard (1983)



# Group speed ( $c_g$ ) and phase speed ( $c_p$ )

- $c_p$  is the speed at which individual wave crests advance
- $c_g$  is the speed at a wave group advances and the speed at which energy is transmitted by a wave
- For a dispersive wave,  $c_g \neq c_p$
- $c_p$  is determined by the definition  $c_p = \sigma/k$
- $c_g = \partial\sigma/\partial k$  and is determined by the dispersion relation
- For deepwater waves  $c_g = c_p/2$
- For shallow water waves (which are non-dispersive)  $c_g = c_p$

Individual wave crests move faster than the group and advance through the wave group



$C_p$ , phase speed =  $\sigma/k$   
 $C_g$ , group speed =  $\partial\sigma/\partial k$

Image courtesy of Wikipedia

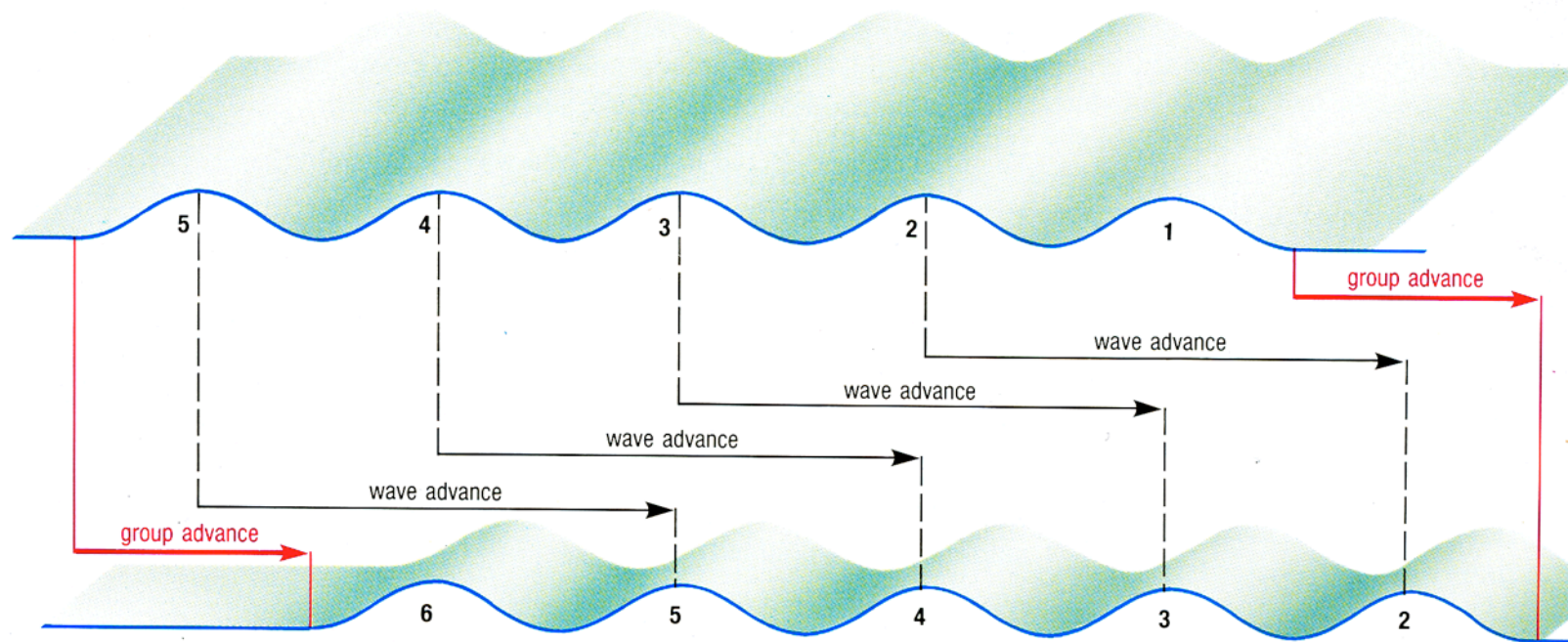


Fig 1.10 WTS

## Group speed for deepwater waves

$$\sigma = \sqrt{gk}$$

$$c_g = \frac{\partial \sigma}{\partial k} = \frac{1}{2} (g/k)^{1/2}$$

$$\frac{c_g}{c_p} = \frac{1}{2} (g/k)^{1/2} (k/g)^{1/2} = \frac{1}{2}$$

## Group speed for shallow water waves

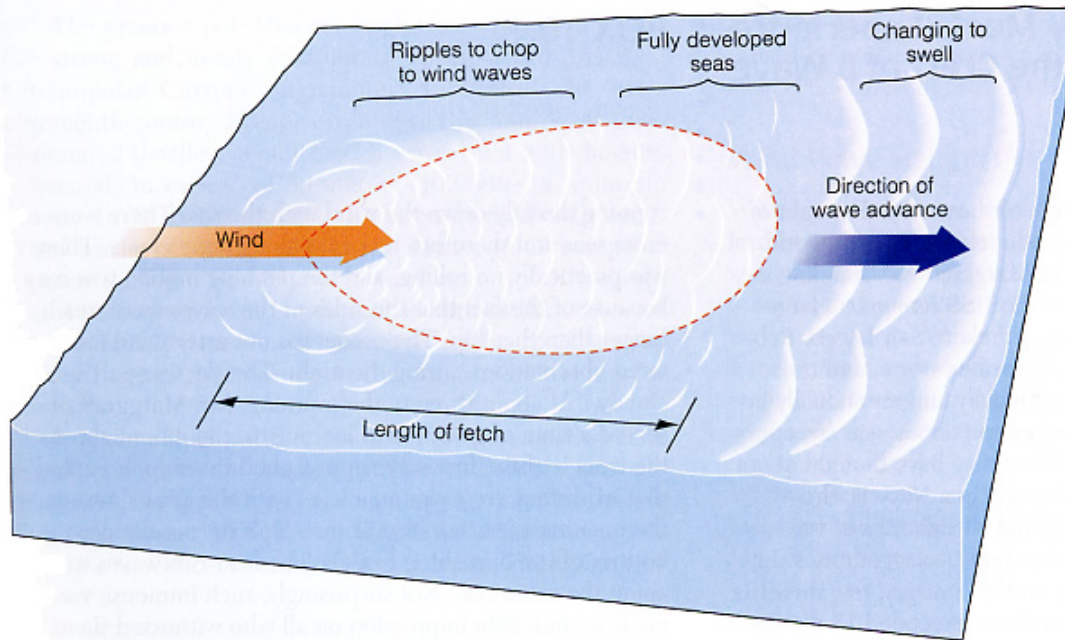
$$\sigma = k\sqrt{gd}$$

$$c_g = \frac{\partial \sigma}{\partial k} = (gd)^{1/2}$$

$$c_p = (gd)^{1/2} \longrightarrow \frac{c_g}{c_p} = 1$$



# Growth of Wind Waves

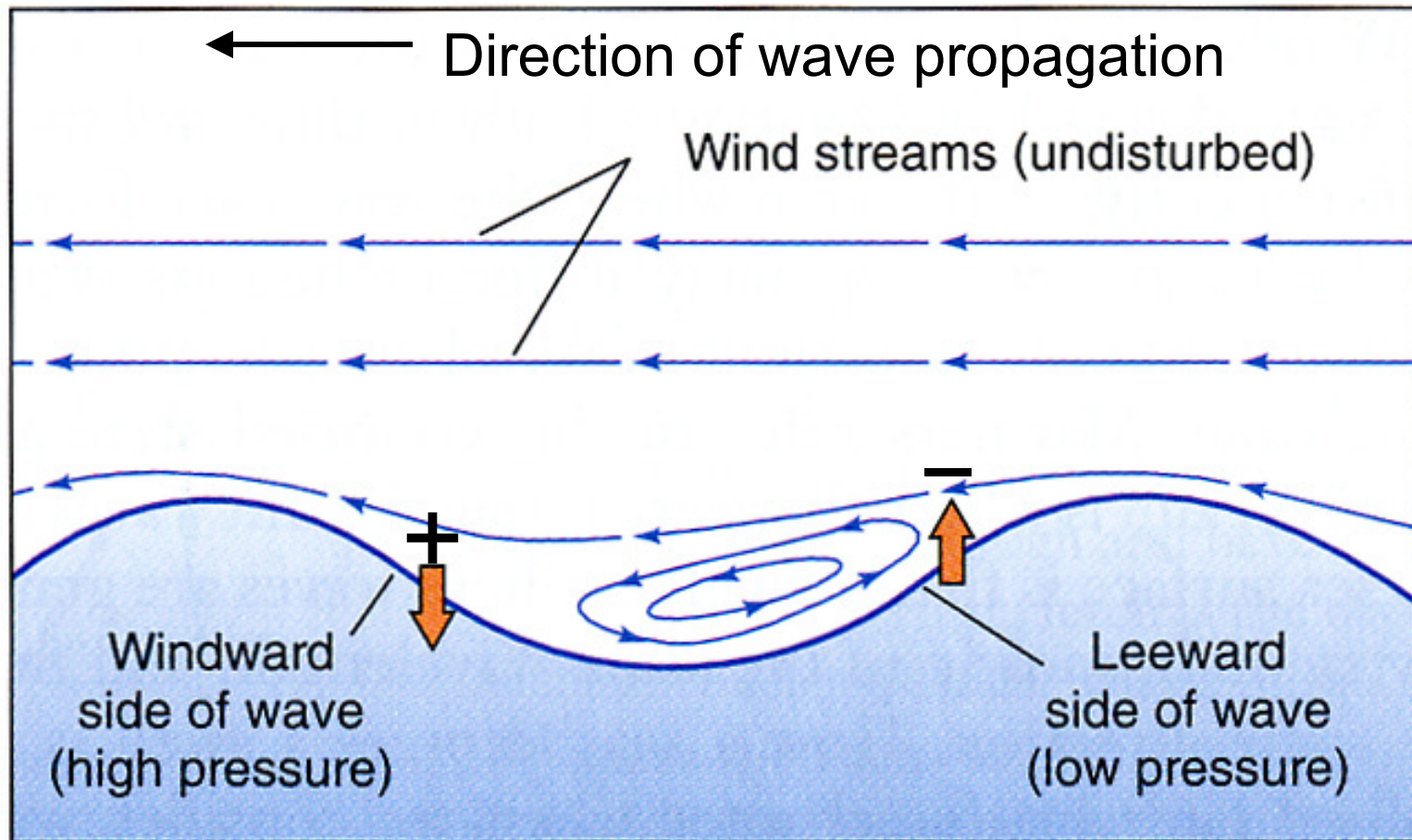


Note: a detailed examination of wave generation is beyond the scope of this class.

**Figure 10.11** The fetch, the uninterrupted distance over which the wind blows without significant change in direction. Wave size increases with increased wind speed, duration, and fetch. A strong wind must blow continuously for three days for the largest waves to fully develop.

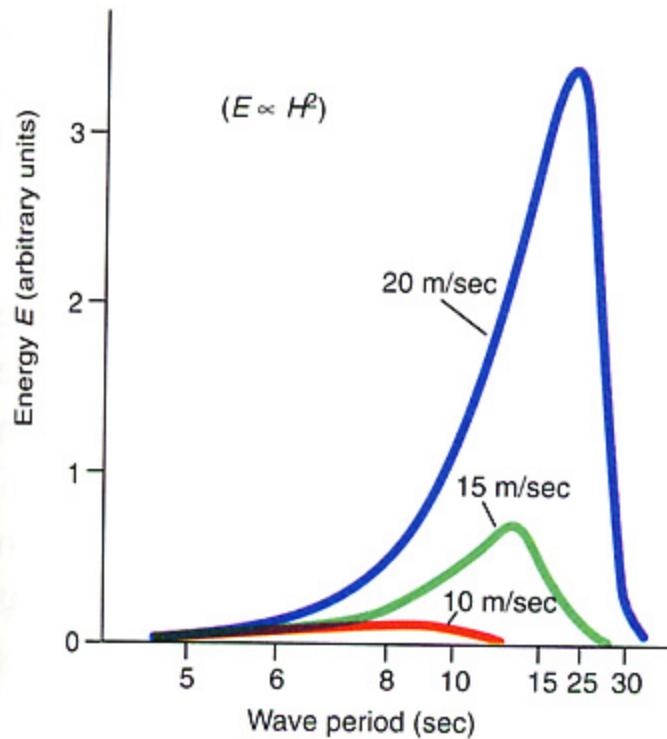
- Wind factors controlling wave growth:
  - strength
  - duration
  - fetch (distance over which the wind blows unimpeded across the water)

## Jeffreys' sheltering model of wave generation

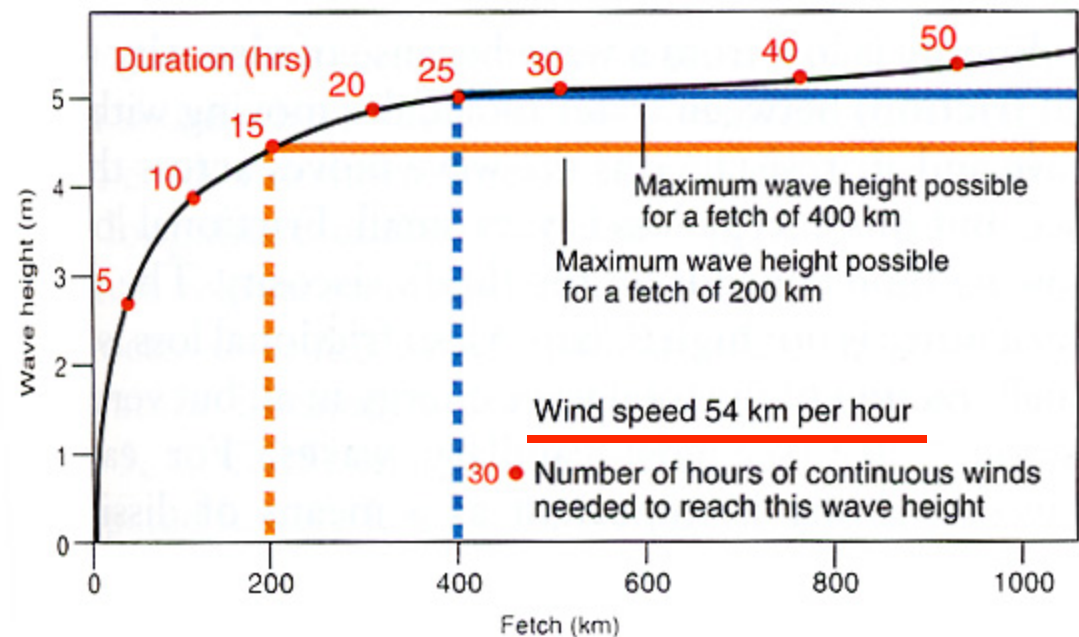


# Fully-Developed Sea

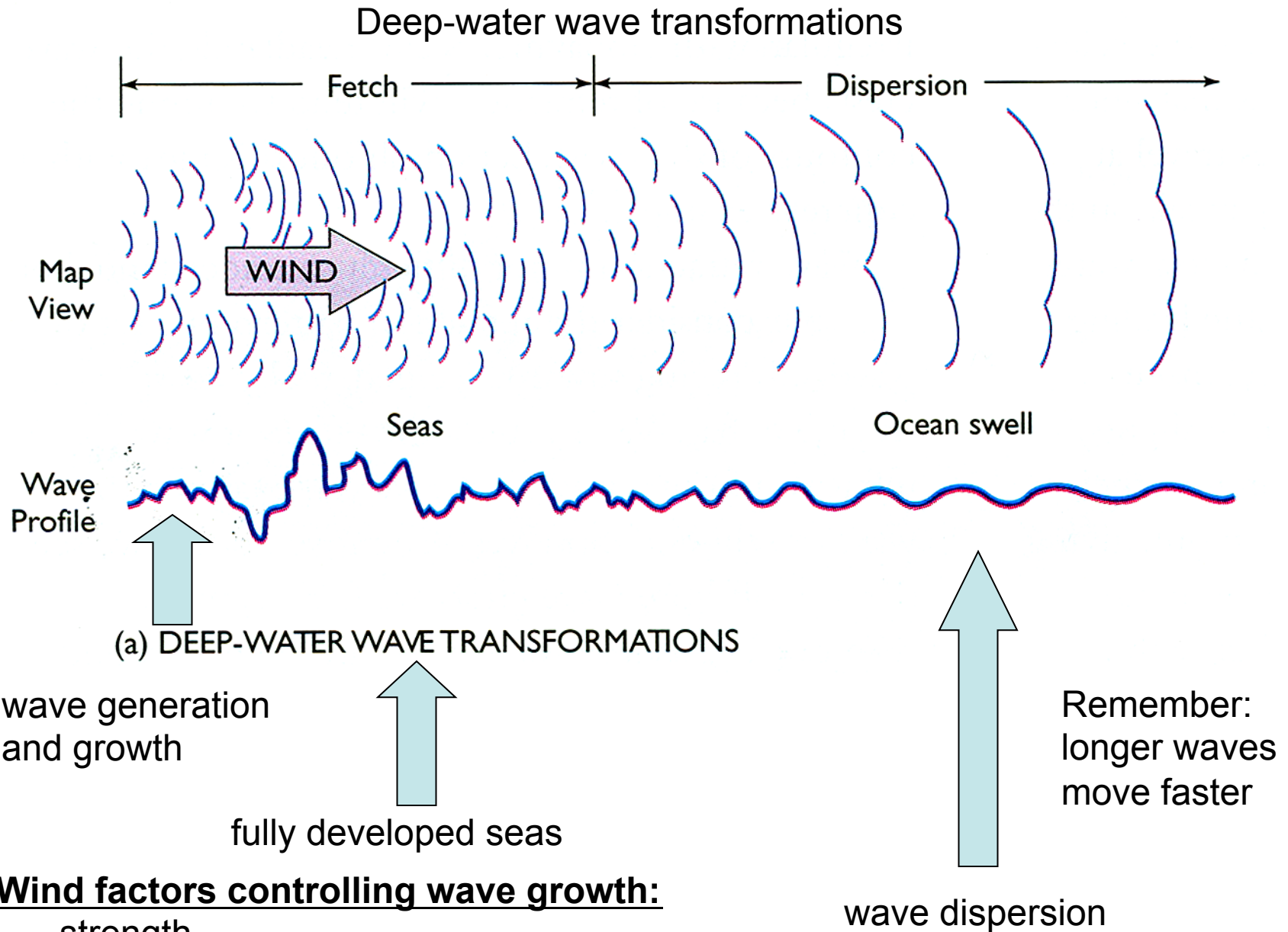
Ocean waves so energetic that input from wind equals losses to wave breaking.



Compare to 16.7, IPO



- Stronger winds = more energy, longer T  
Duration to fully developed is about ½ to 1 day or so  
(depends on wind speed and fetch)

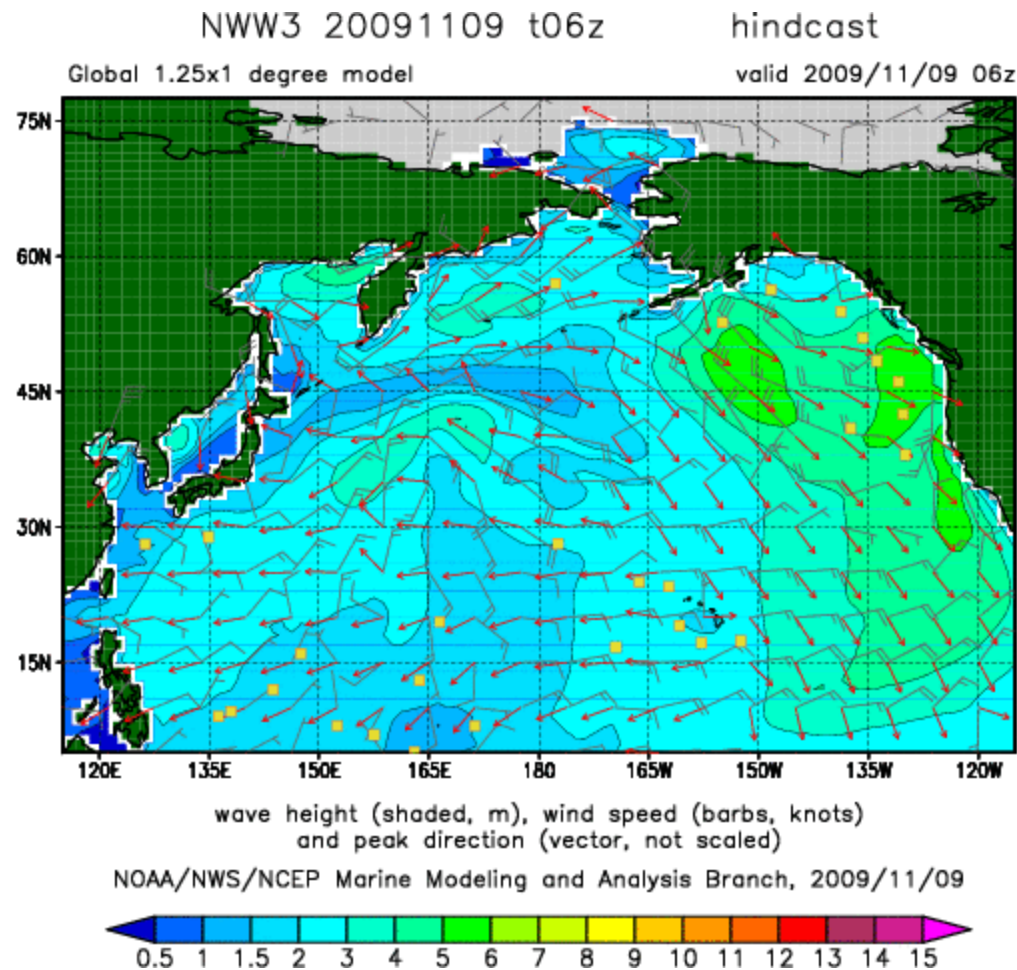


- **Wind factors controlling wave growth:**

- strength
- duration
- fetch (distance over which the wind blows unimpeded across the water)



In practice, wave forecasts are based on heavily empirical formulas informed by meteorological models of winds



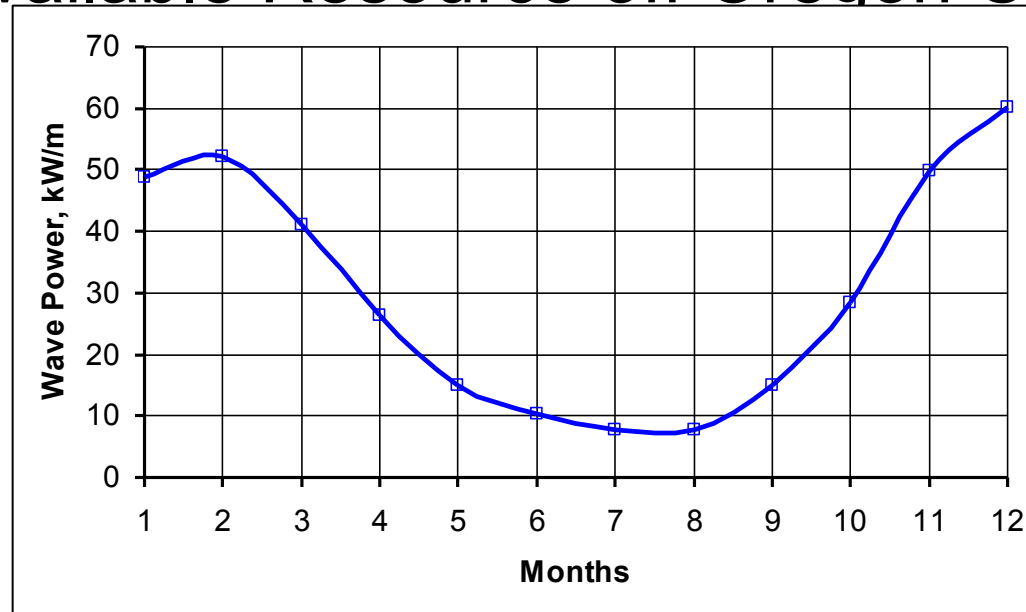
Model assumptions OK for spatial scales larger than 1-10 km

Can reach this link from NOAA WW3 page  
[https://www.fnmoc.navy.mil/ww3\\_cgi/index.html](https://www.fnmoc.navy.mil/ww3_cgi/index.html)

# Power from Ocean Waves

## Available Resource off Oregon Coast

Seasonal variation –  
Good match  
for the NW load  
demand



Data buoys are  
2-200mi off shore,  
with waves  
traveling 15-20mph,  
gives 10+ hours  
forecast time for  
buoy generators  
located 2 mi out

(wave data From National Data Buoy Center, Power estimated from  
5 buoys off the Oregon coast over past 10 years)

Power from a wave is  $P = \frac{\rho g^2 T H^2}{32\pi}$  W/m of crest length (distance along an individual crest)

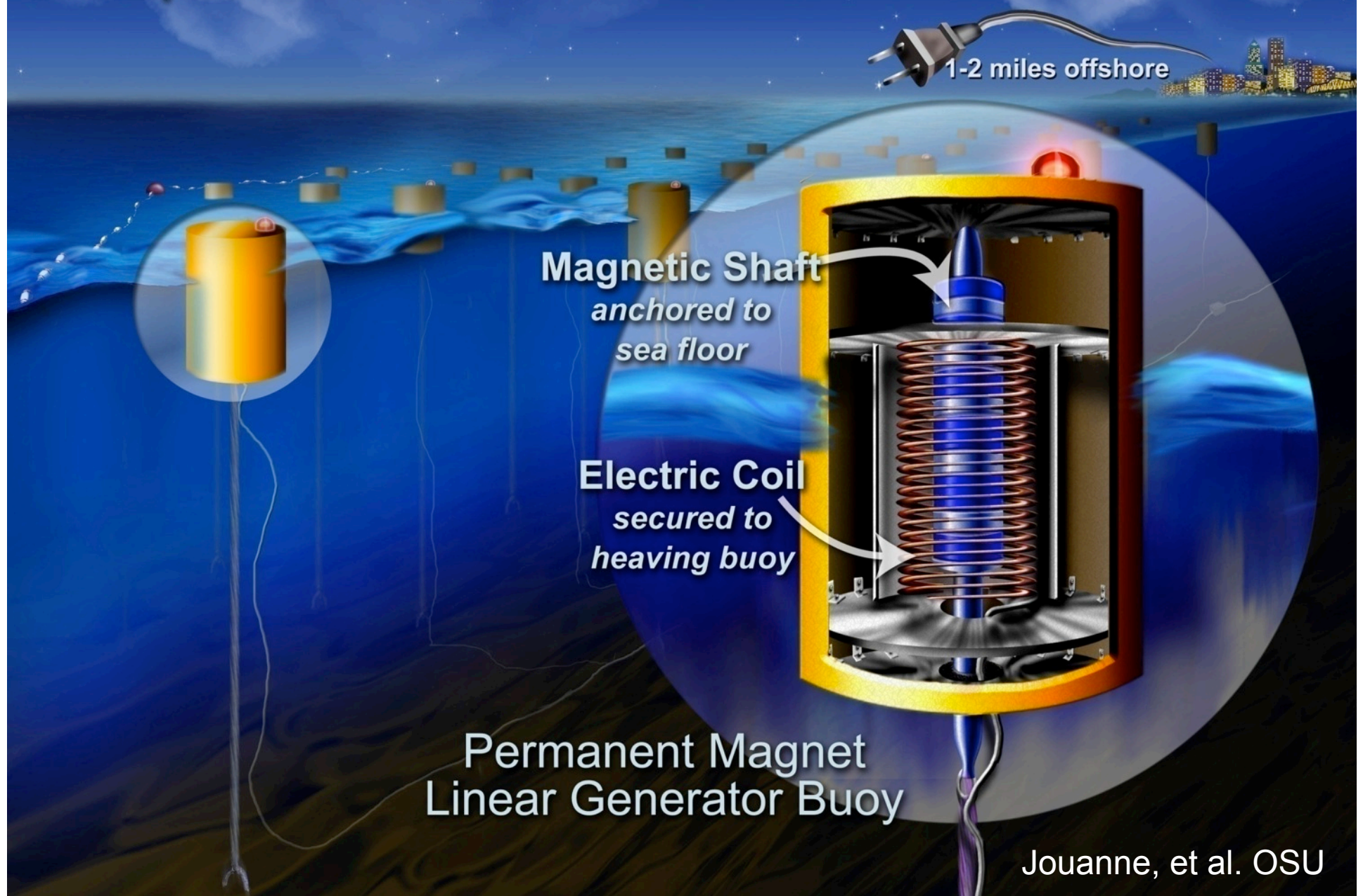
$\rho$  = the density of sea water = 1025 kg/m<sup>3</sup>

$g$  = acceleration due to gravity = 9.8 m/s<sup>2</sup>

$T$  = period of wave (s) (averages 8s in the winter to 6s in the summer)

$H$  = wave height (m) (averages 3.5m in the winter to 1.5m in the summer)

# Oregon State University Conceptual Wave Park



Jouanne, et al. OSU