

**Numerical Simulations of the 2004 Indian Ocean Tsunami –  
Runup and Inundation**

**Xiaoming Wang and Philip L.-F. Liu  
Cornell University**

WHOI Workshop  
Interactions between Tsunamis and Underwater Geological Processes  
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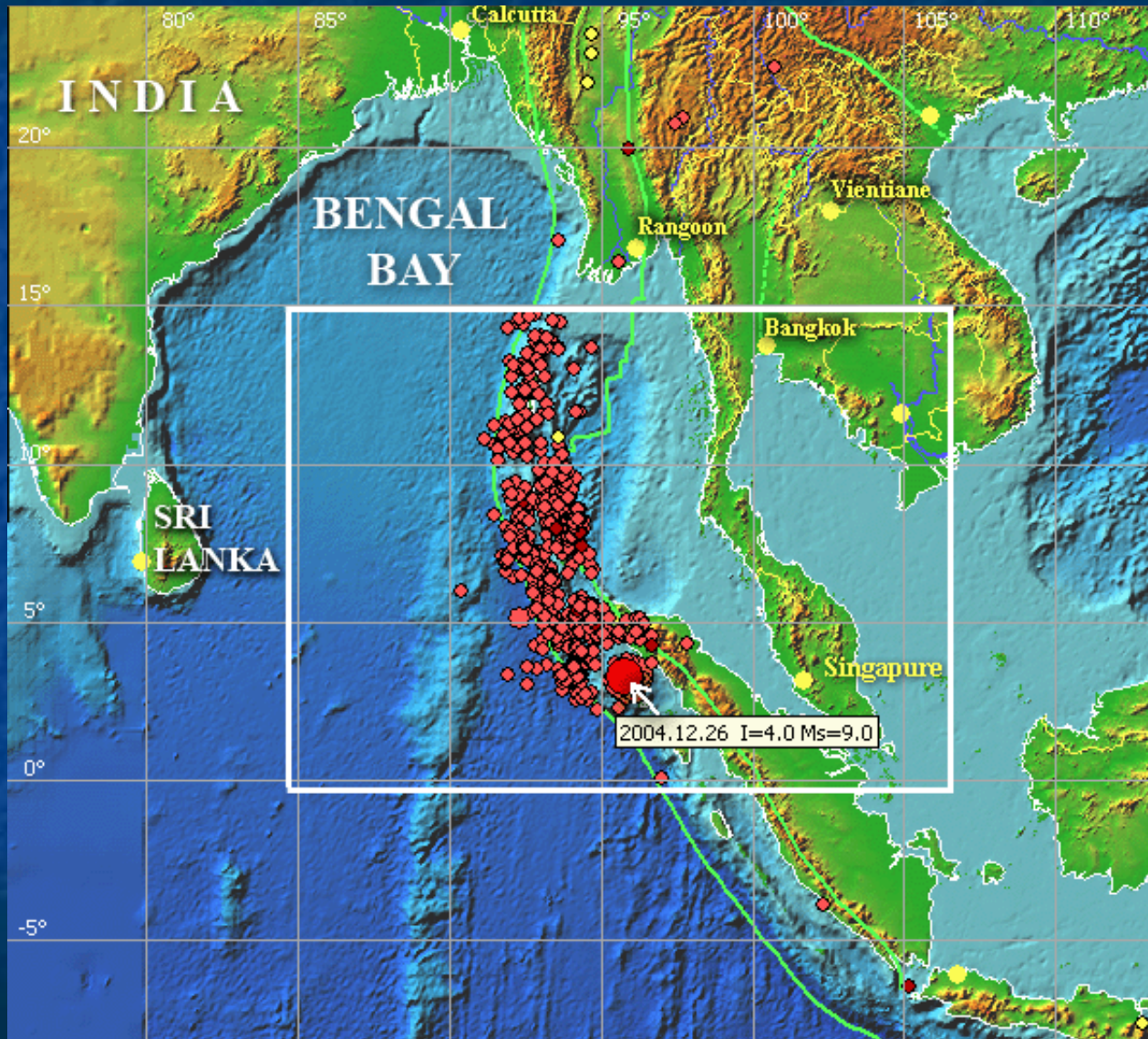
# **Outline:**

- 1. Review of the general features of the 2004 Indian Ocean tsunamis**
- 2. Numerical simulation results of runup and inundation in Trincomalee, Sri Lanka and Banda Aceh, Indonesia**
- 3. Erosion and deposit**



# Epicenter and after shocks

## 2004 Sumatra Earthquake

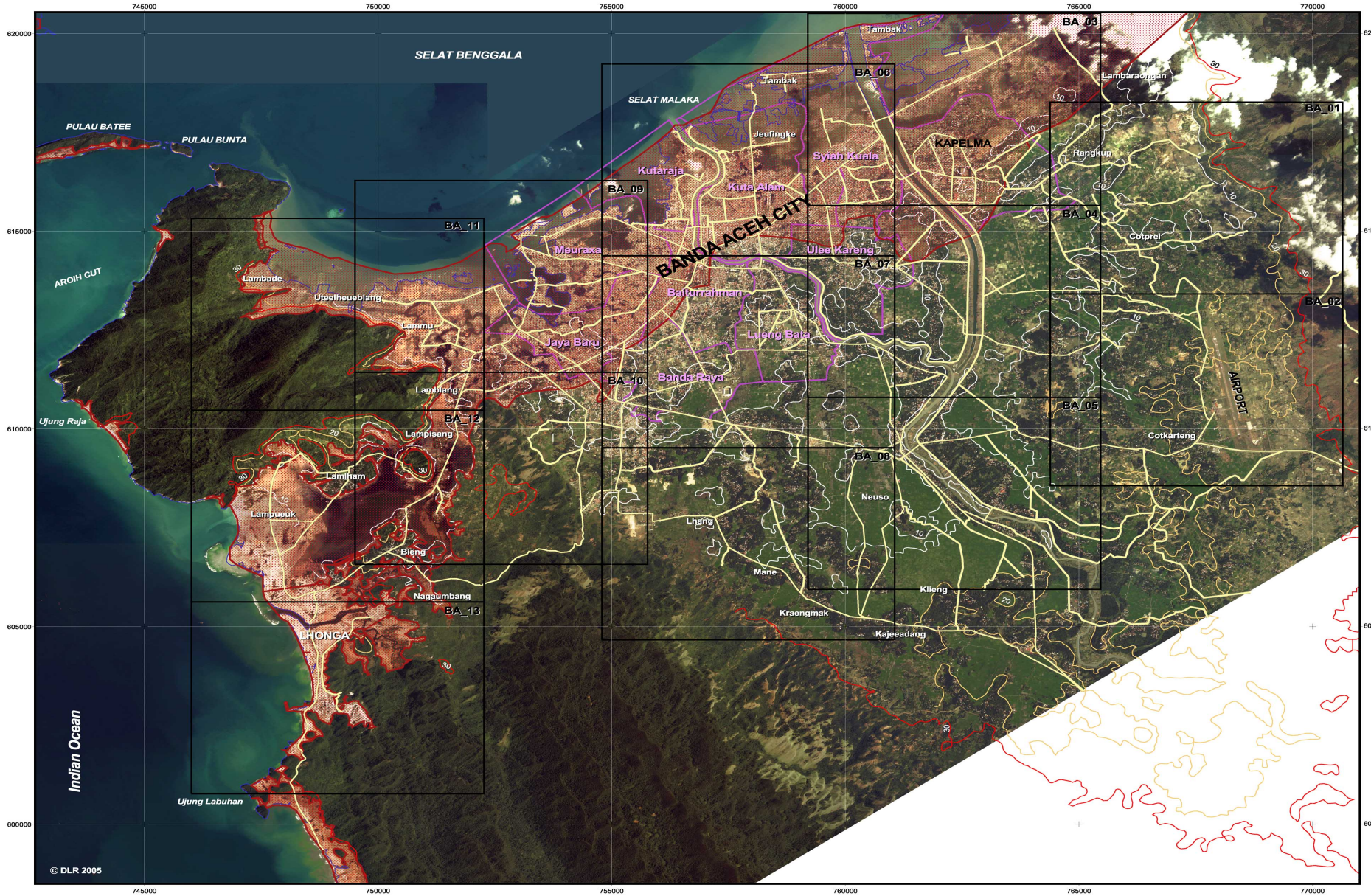




# Flooding and erosion at Banda Aceh

INDONESIA/SUMATRA - Banda Aceh Region

1 : 45.000





Jan 23 2004



Banda Aceh North Shore

Dec 28 2004





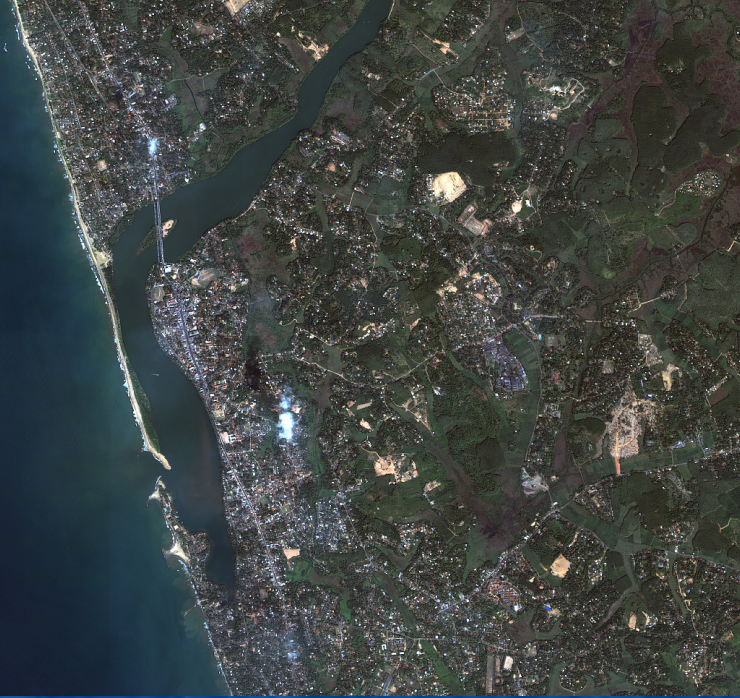
# Southern Banda Aceh (Gleebruk: 31miles southwest of Banda Aceh)





January 1, 2004

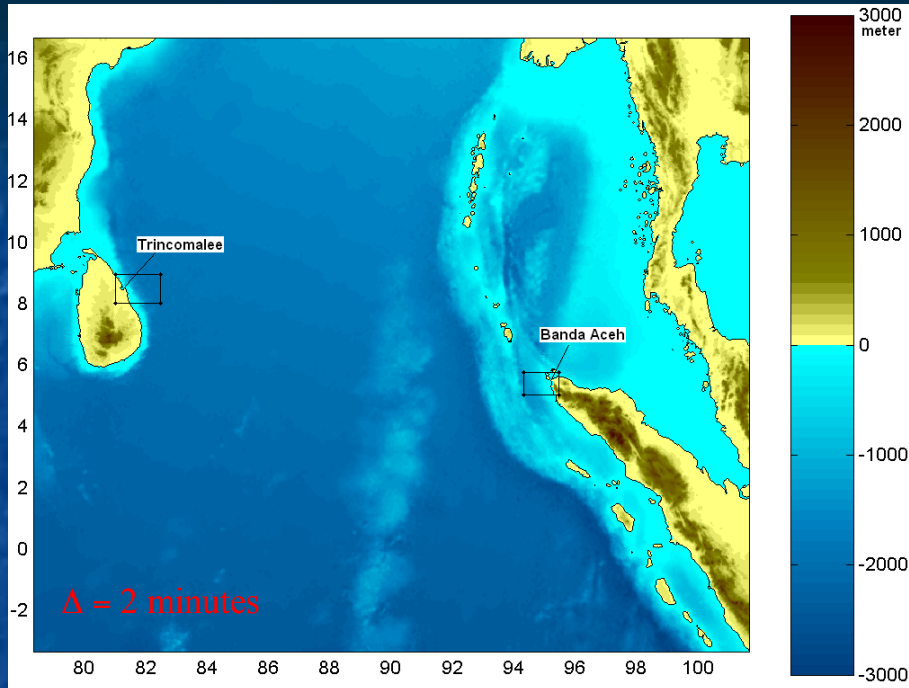
Kalutara, Sri Lanka



December 26, 2005



# COMCOT: Nested grid system

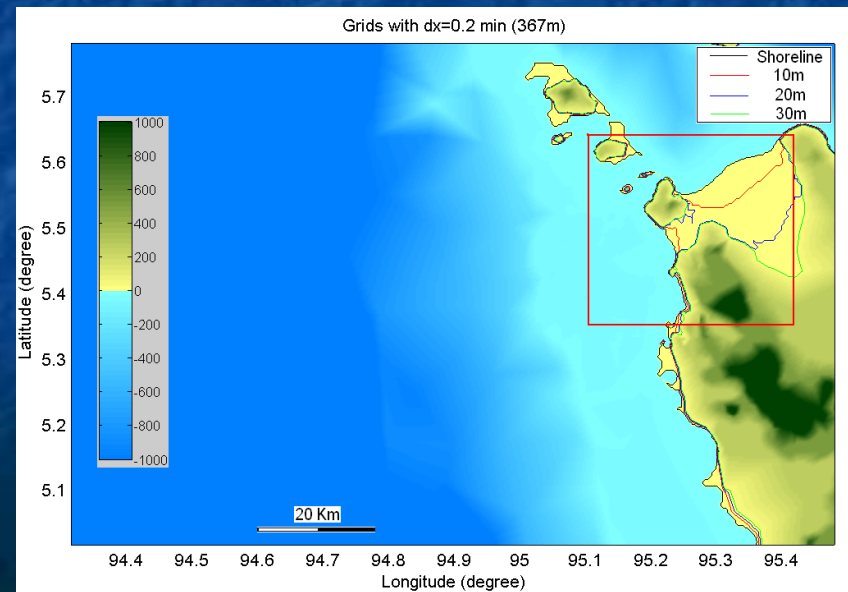
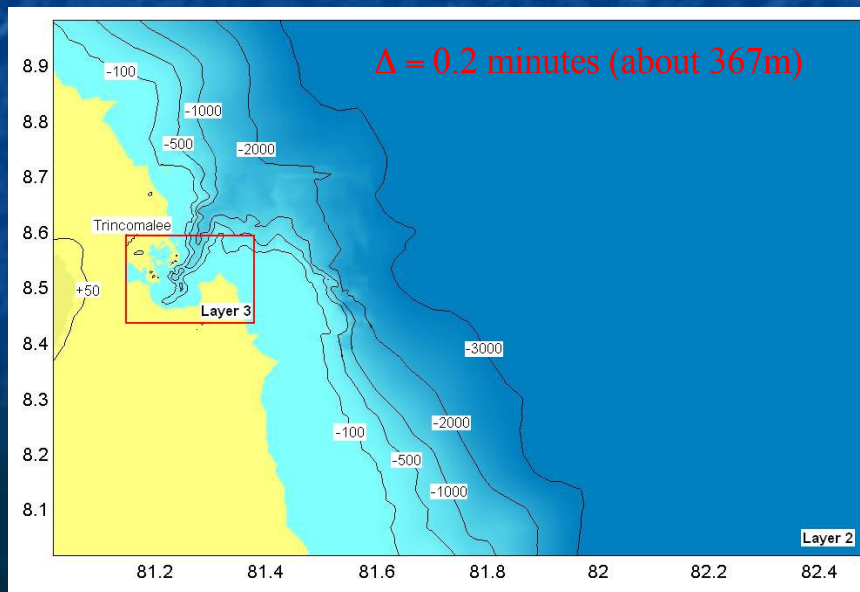


## Linear Shallow Water Equations in Spherical Coordinates:

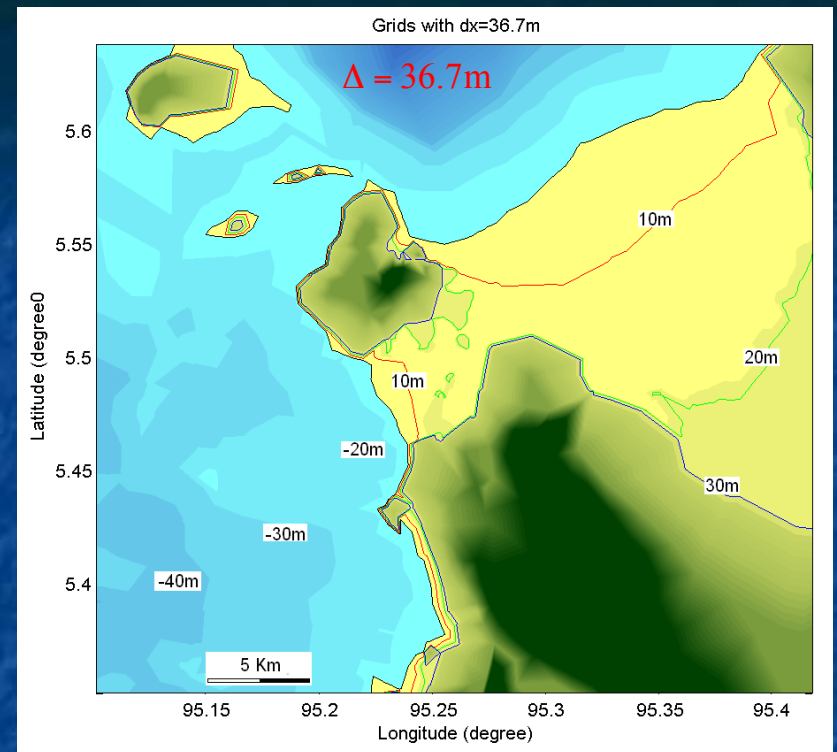
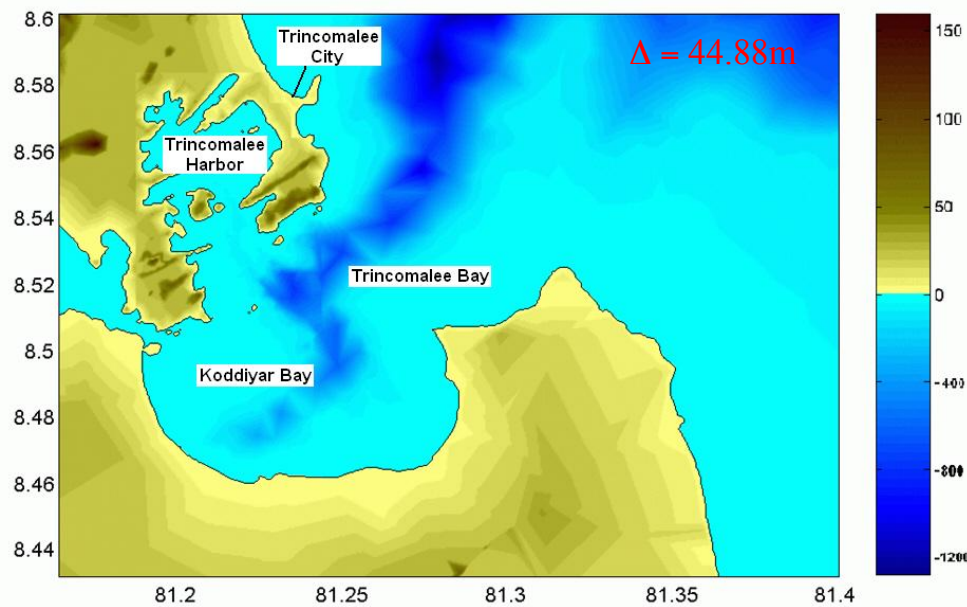
$$\frac{\partial \zeta}{\partial t} + \frac{1}{R \cos \varphi} \left[ \frac{\partial P}{\partial \psi} + \frac{\partial}{\partial \varphi} (\cos \varphi Q) \right] = 0$$

$$\frac{\partial P}{\partial t} + \frac{gh}{R \cos \varphi} \frac{\partial \zeta}{\partial \psi} - fQ = 0$$

$$\frac{\partial Q}{\partial t} + \frac{gh}{R} \frac{\partial \zeta}{\partial \varphi} + fP = 0$$







## Non-linear Shallow Water Equations in Cartesian Coordinates:

$$\frac{\partial \xi}{\partial t} + \frac{\partial P}{\partial x} + \frac{\partial Q}{\partial y} = 0$$

$$\frac{\partial P}{\partial t} + \frac{\partial}{\partial x} \left( \frac{P^2}{H} \right) + \frac{\partial}{\partial y} \left( \frac{PQ}{H} \right) + gH \frac{\partial \xi}{\partial x} + \tau_x H = 0$$

$$\frac{\partial Q}{\partial t} + \frac{\partial}{\partial x} \left( \frac{PQ}{H} \right) + \frac{\partial}{\partial y} \left( \frac{Q^2}{H} \right) + gH \frac{\partial \xi}{\partial y} + \tau_y H = 0$$

## Bottom Frictional stress:

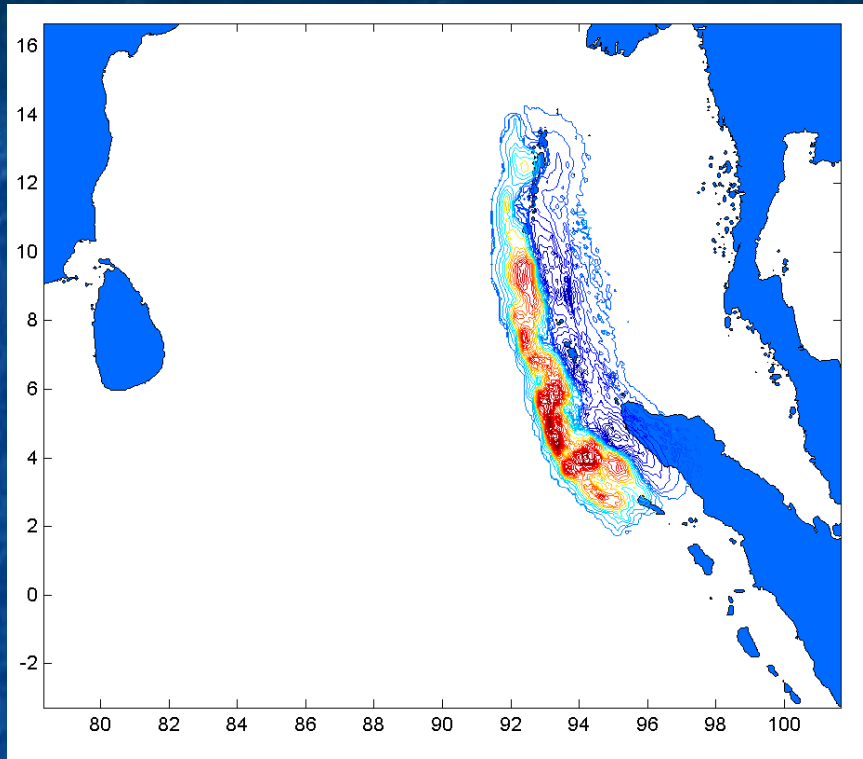
$$\tau_x = \frac{gn^2}{H^{10/3}} P(P^2 + Q^2)^{1/2}$$

$$\tau_y = \frac{gn^2}{H^{10/3}} Q(P^2 + Q^2)^{1/2}$$

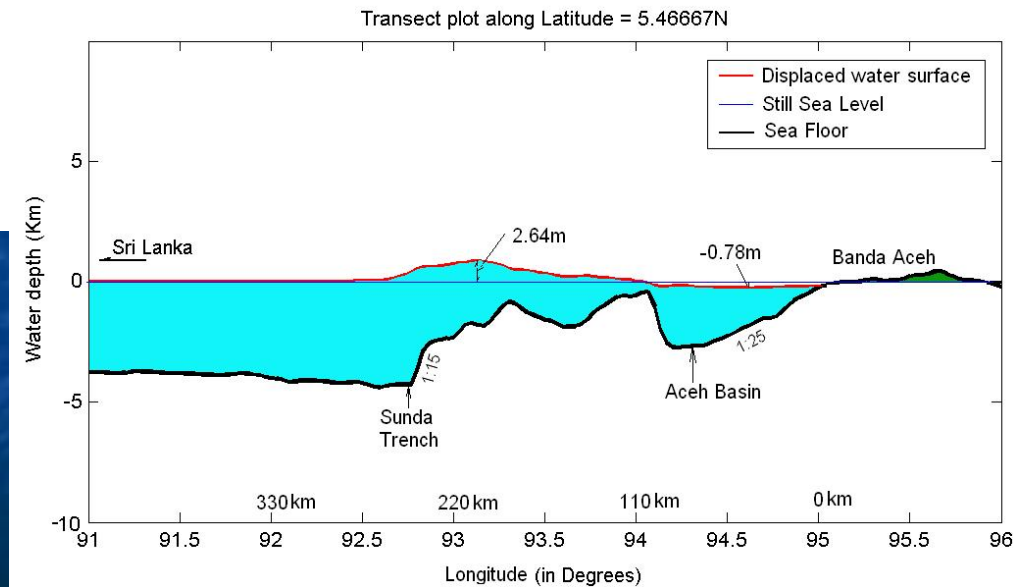
$$n = 0.02$$



# Initial Free Surface Profile

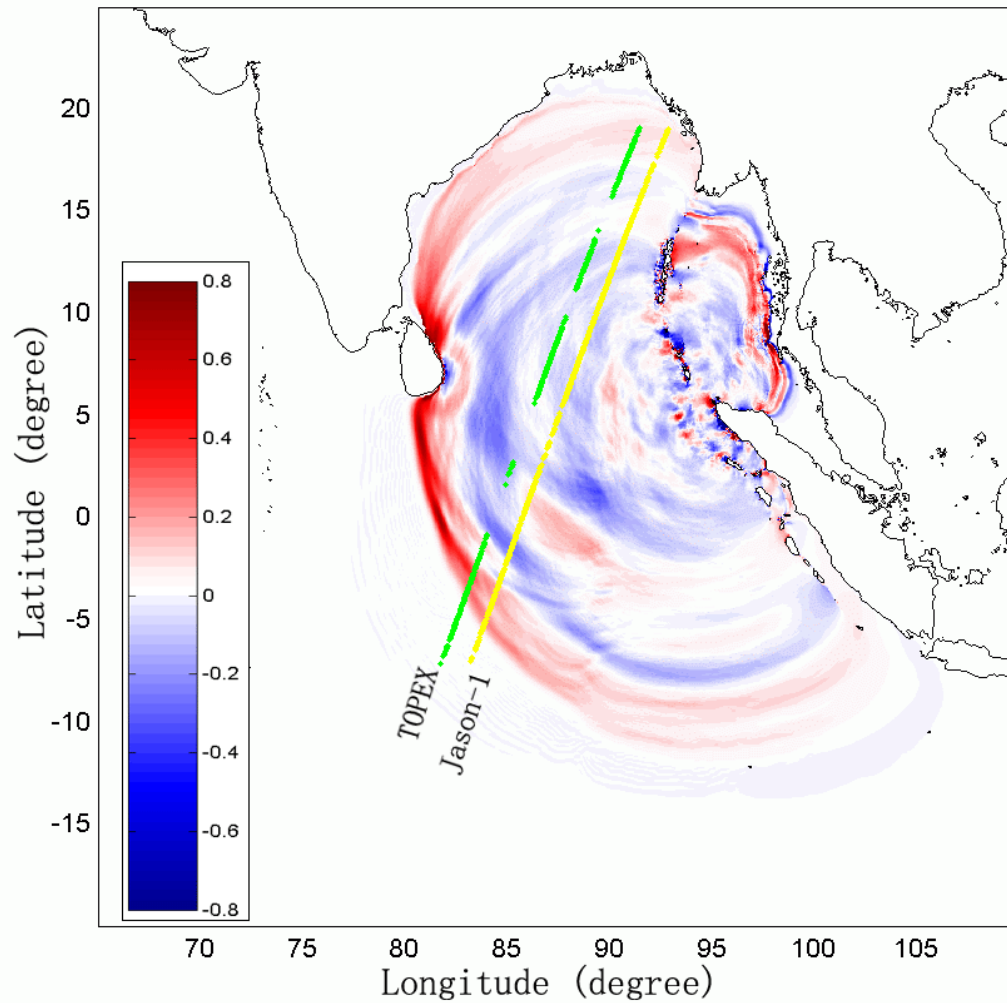


Rupture speed: 2 ~ 3 km/s  
Rupture duration: 10 mins  
Fault Plane Width: 150 ~ 200km  
Maximum horizontal displacement: 20 m  
Maximum vertical displacement: 3 m





## Satellite tracks for TOPEX and Jason-1

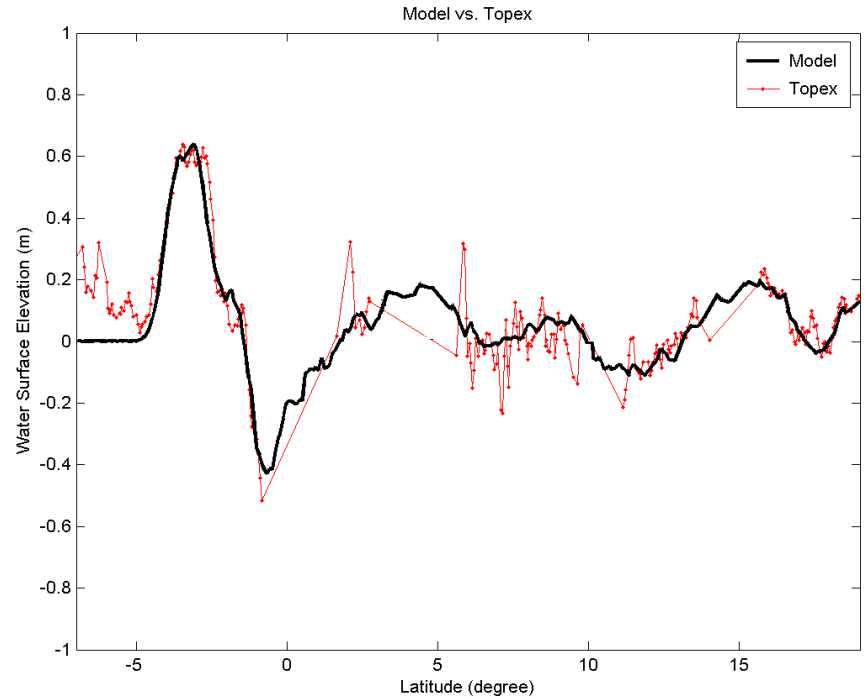
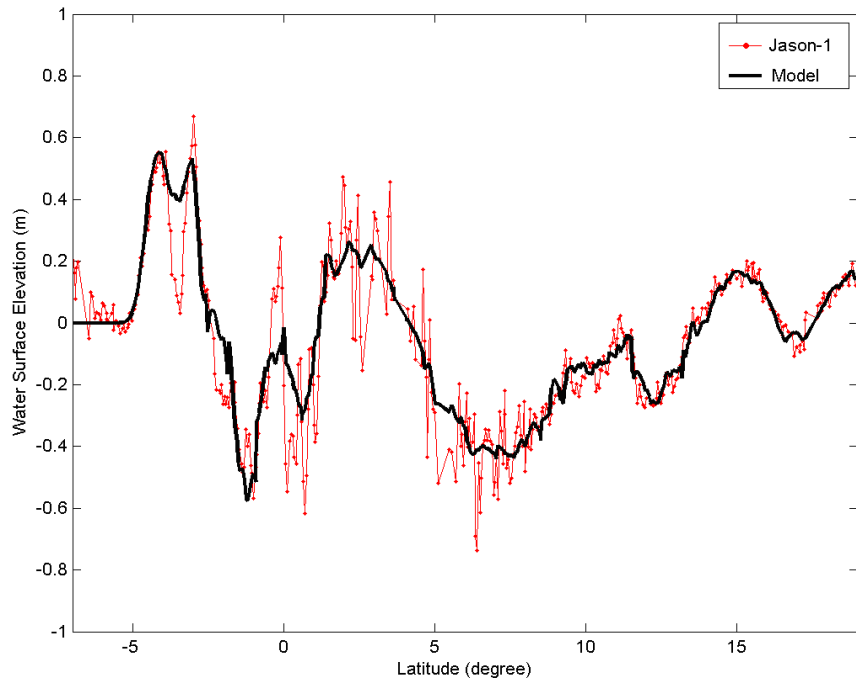


[animation](#)

**The colors indicate the numerically simulated free surface elevation in meter at two hours after the earthquake struck**

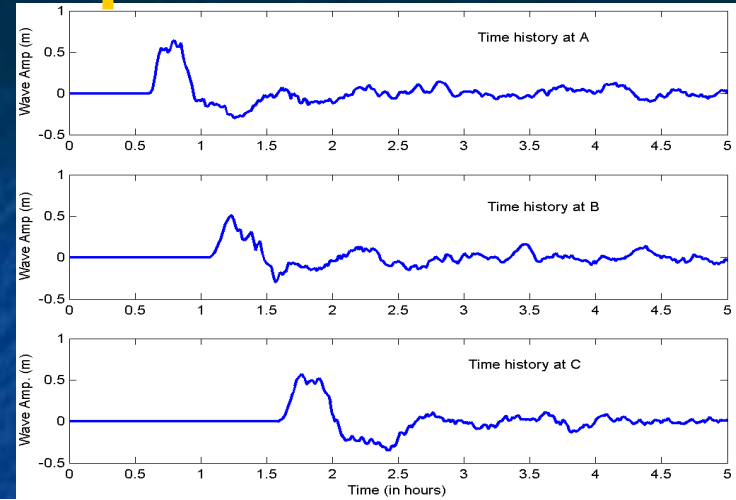
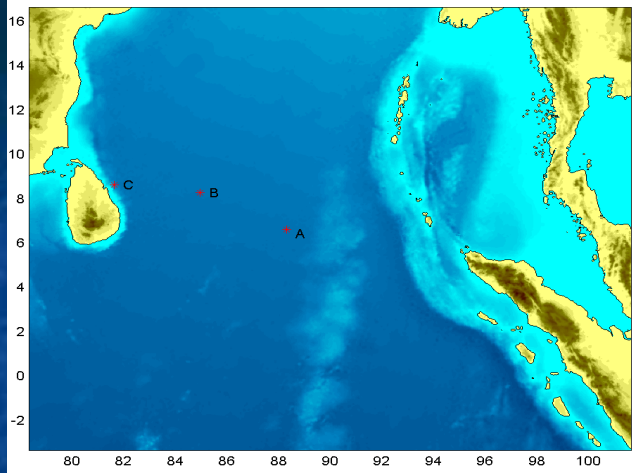


# Comparisons between model results and Jason-1 measurements (left) and TOPEX measurements (right)

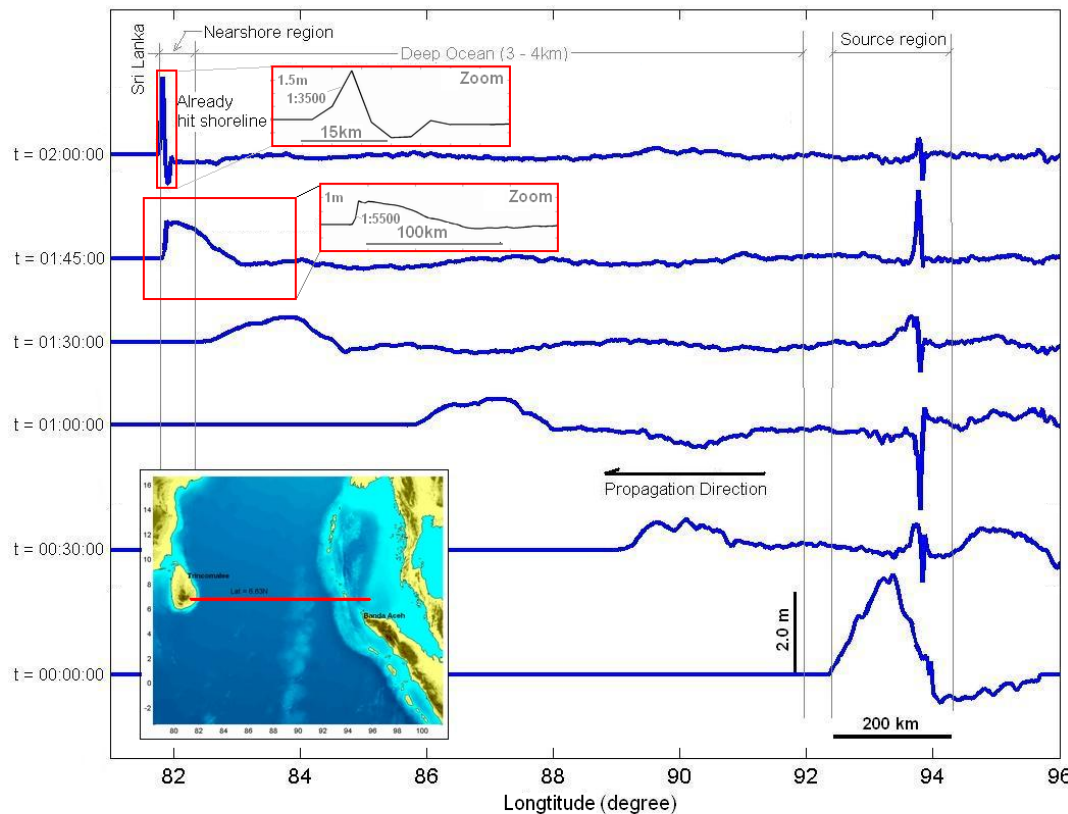




# Tsunami characteristics in the open sea



Cross section plots (along latitude = 6.63°) at different time

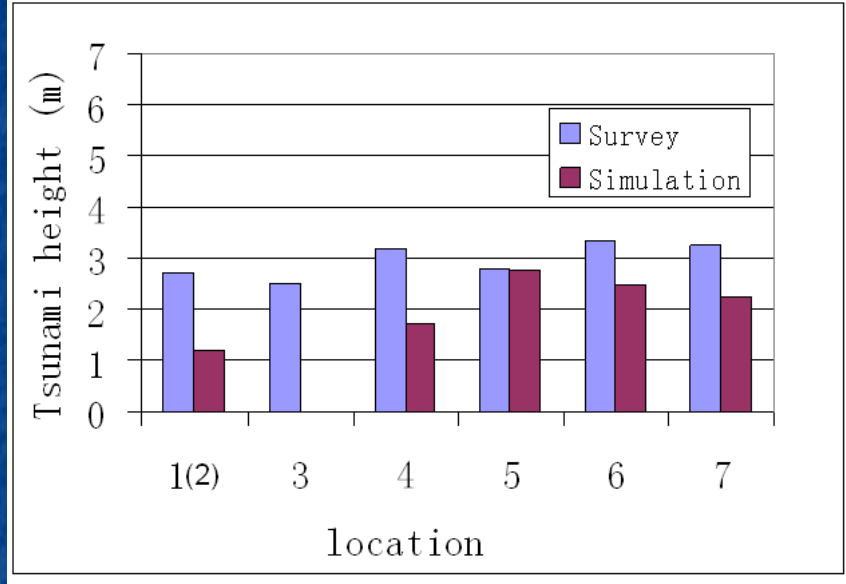
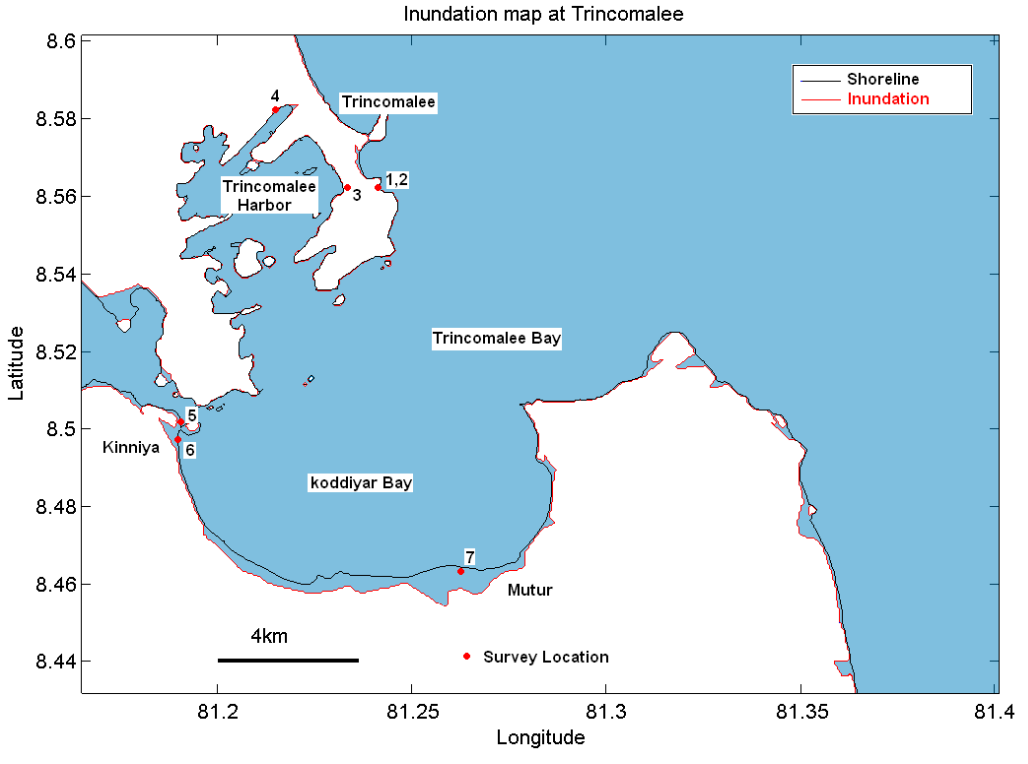


**Snapshots of free surface profile along latitude = 6.63)**

Linear Non-dispersive Waves



# Tsunami inundation in Trincomalee (red line shows the inundation line)



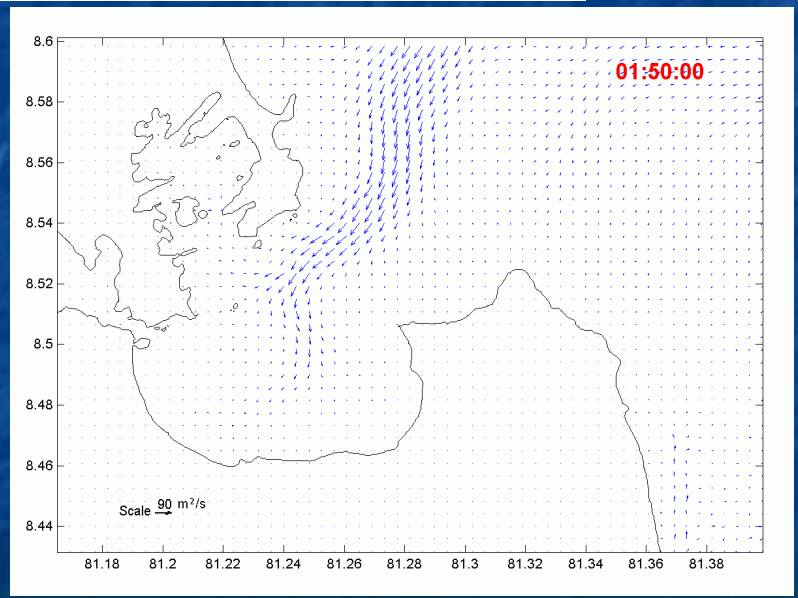
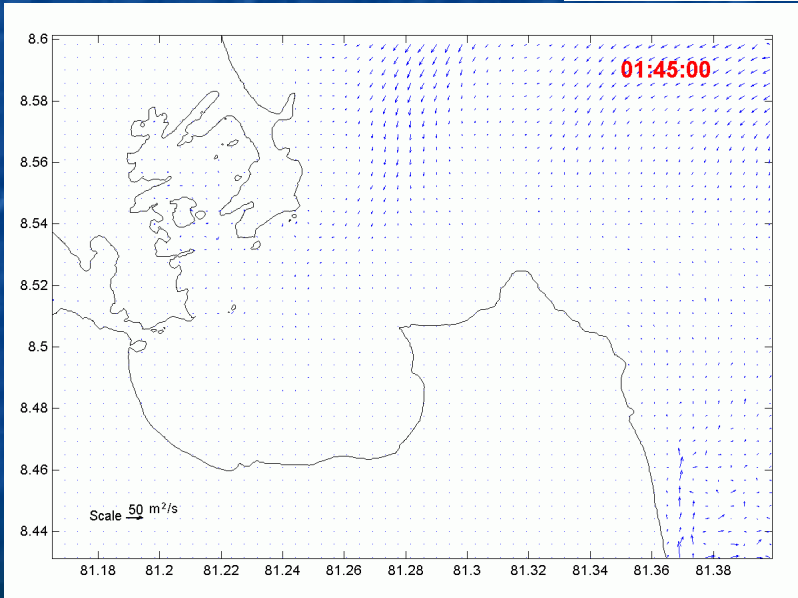
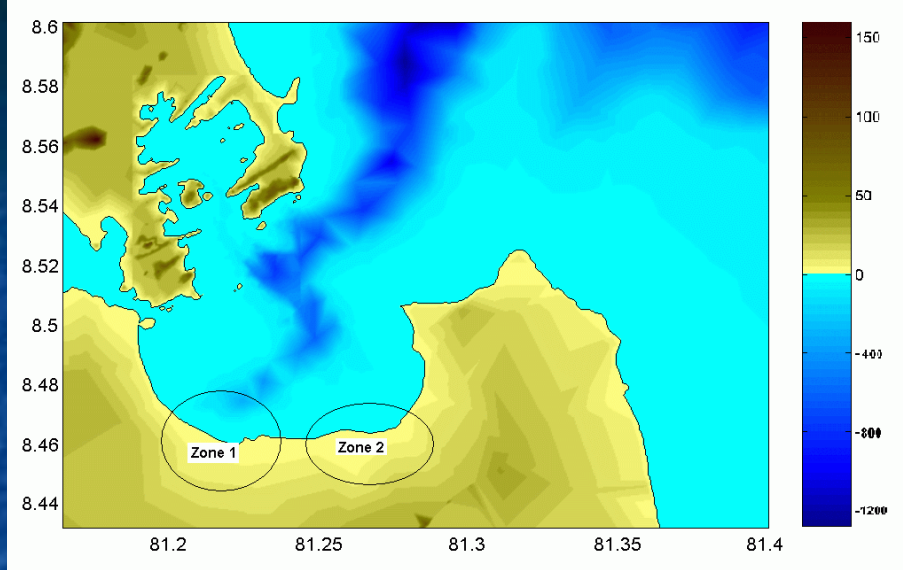
[Wave profile](#)

[animation](#)

Comparisons between survey data and numerical results



# Local Bathymetry Effect

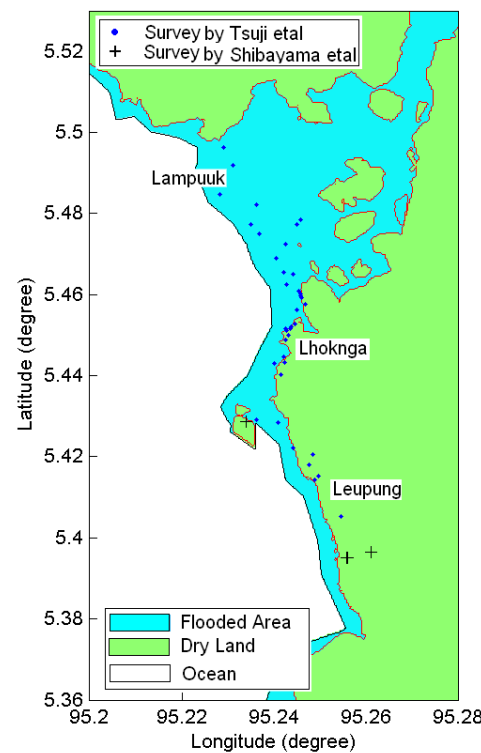
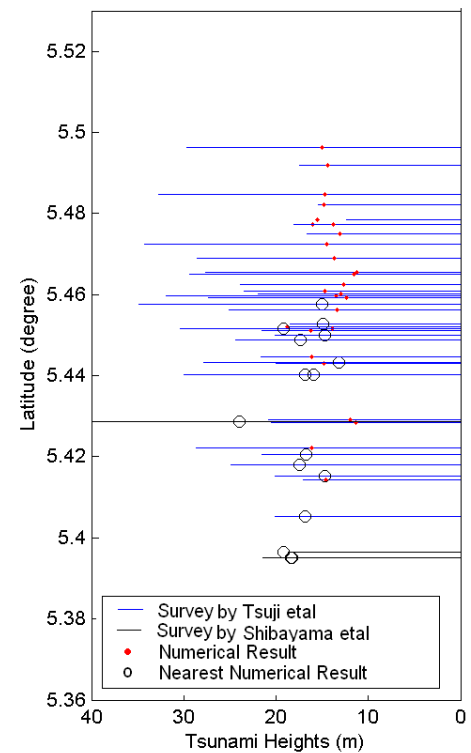
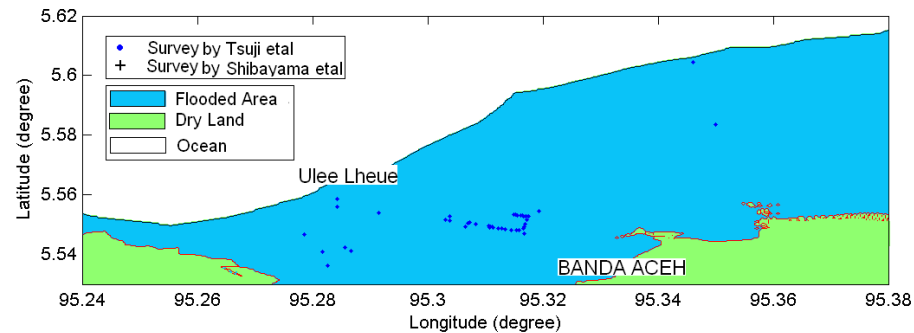
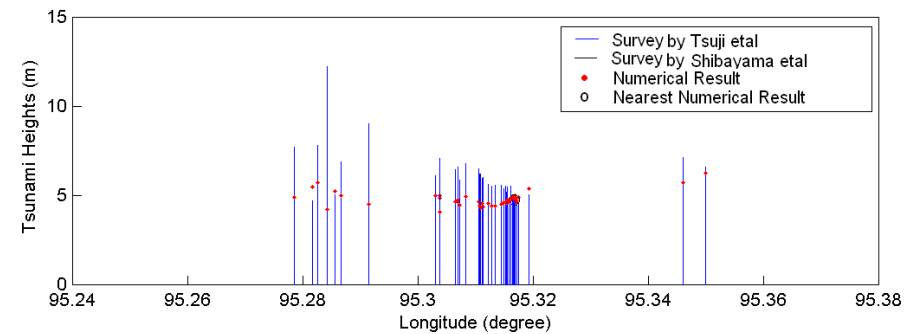


# Mass fluxes inside Trincomalee bay





# Calculated tsunami heights at the Surveyed locations



# Sediment transport

## Shield parameter

$$\theta = \frac{\tau_b}{(\rho_s - \rho)gd_s},$$

$$\tau_b = \sqrt{\tau_x^2 + \tau_y^2}$$

$$\tau_x = \frac{gn^2}{H^{10/3}} P(P^2 + Q^2)^{1/2}$$

$$\tau_y = \frac{gn^2}{H^{10/3}} Q(P^2 + Q^2)^{1/2}$$

$\theta > 0.06$ : incipience of grain movement

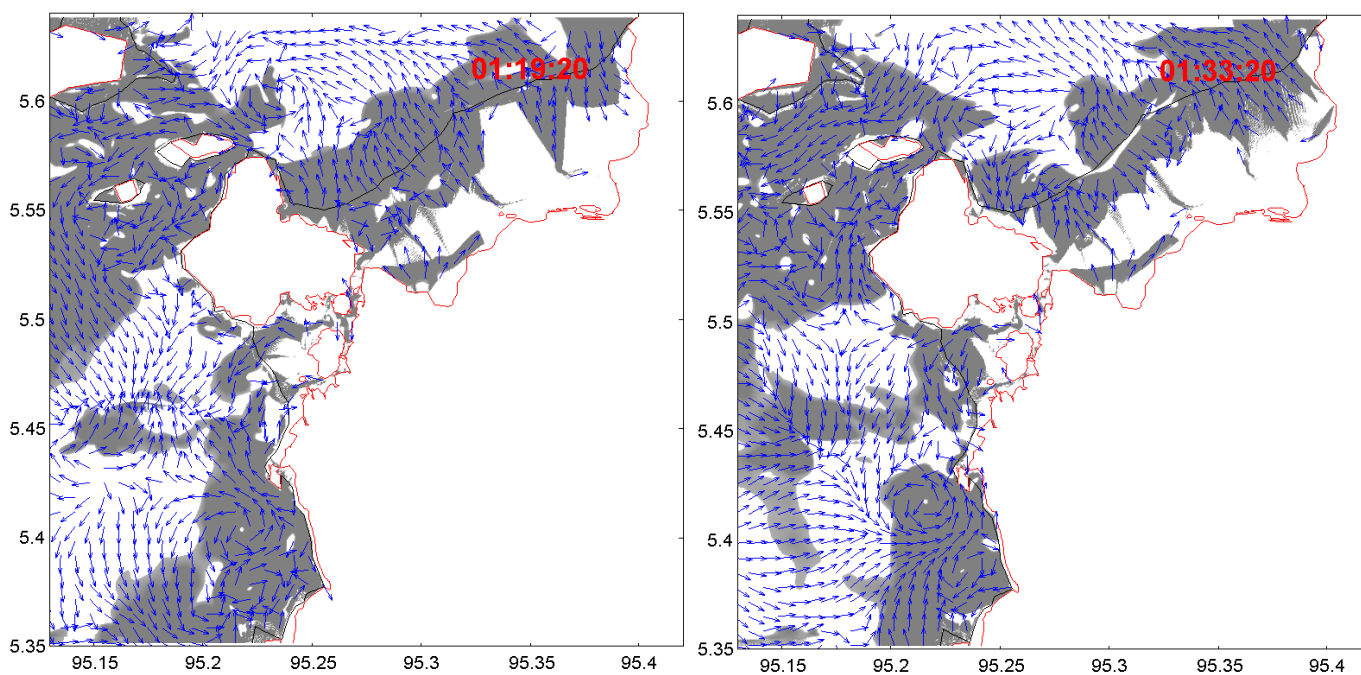
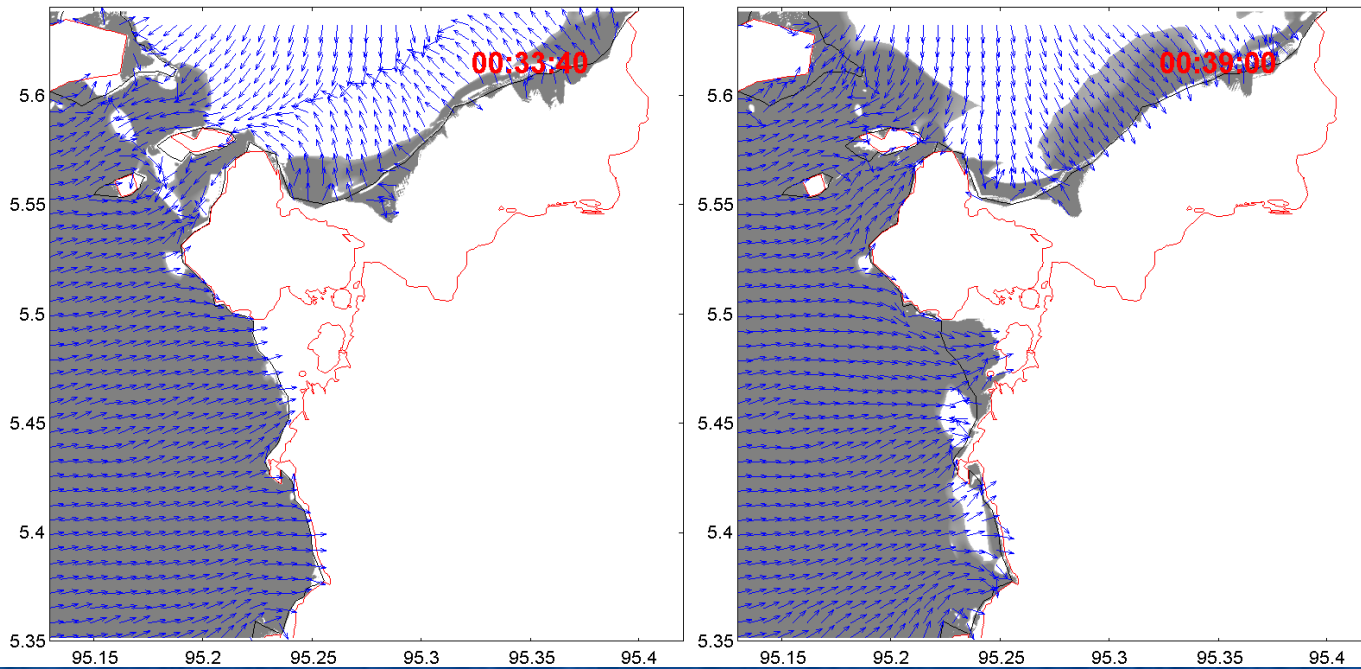


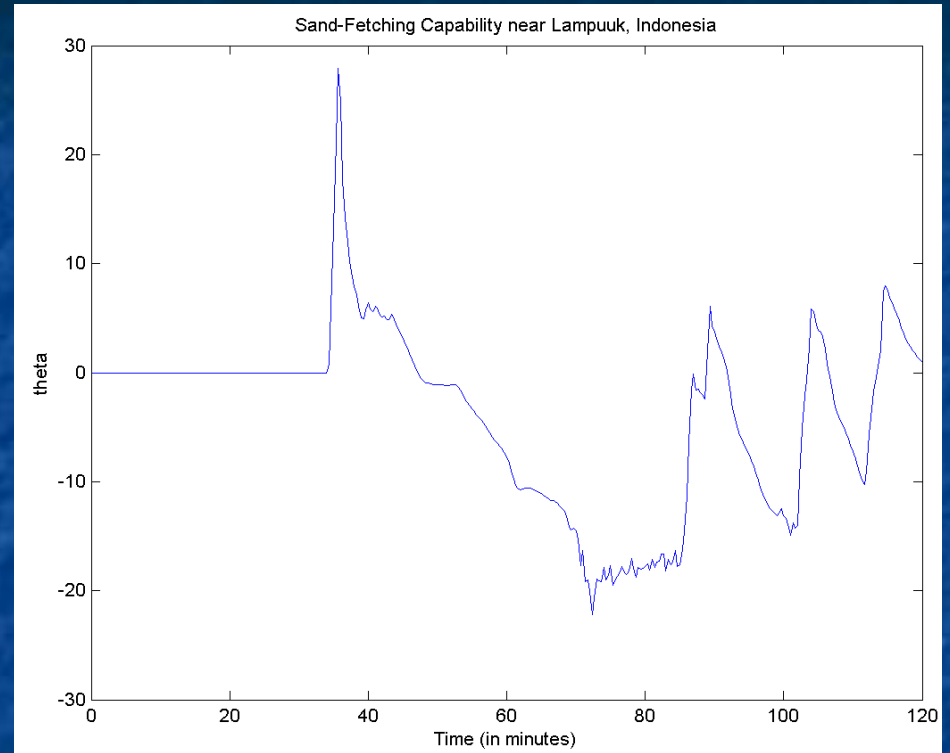
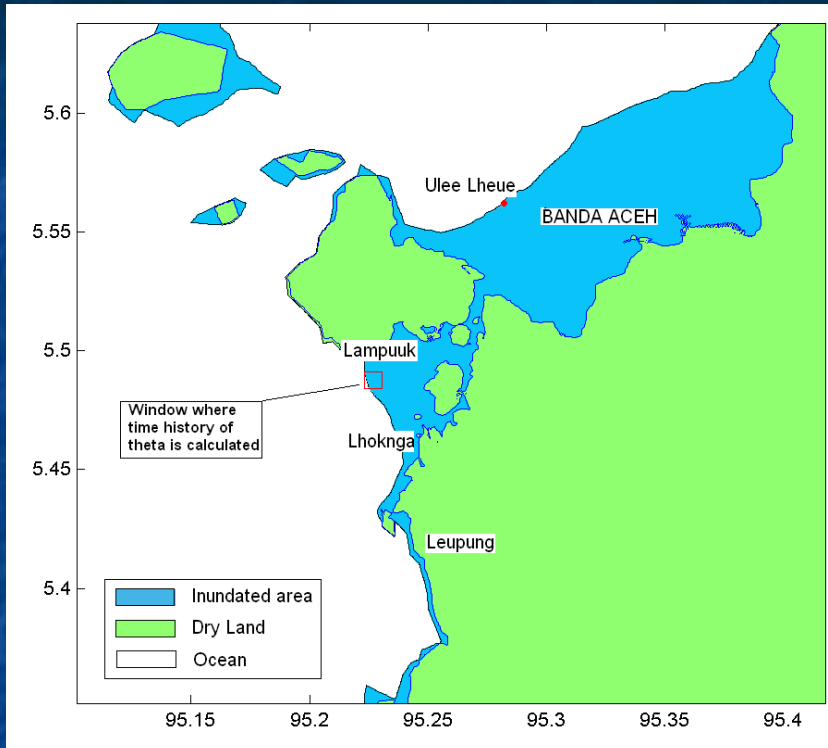
## Sediment movement potential around Banda Aceh

Gray color stands for regions where  $\theta > 0.06$

Arrows denote the direction of bottom shear stress.

The clock shows the time after the main shock





**Time history of  $\theta$  averaged within an 800m-by-800m window near Lampuuk. Positive value means that the flow is in onshore direction and negative value means that the flow is in offshore direction.**



# Sediment erosion and deposit

## Mass conservation

$$\frac{\partial h}{\partial t} + \frac{1}{1-\lambda} \nabla \cdot \mathbf{q} = 0$$

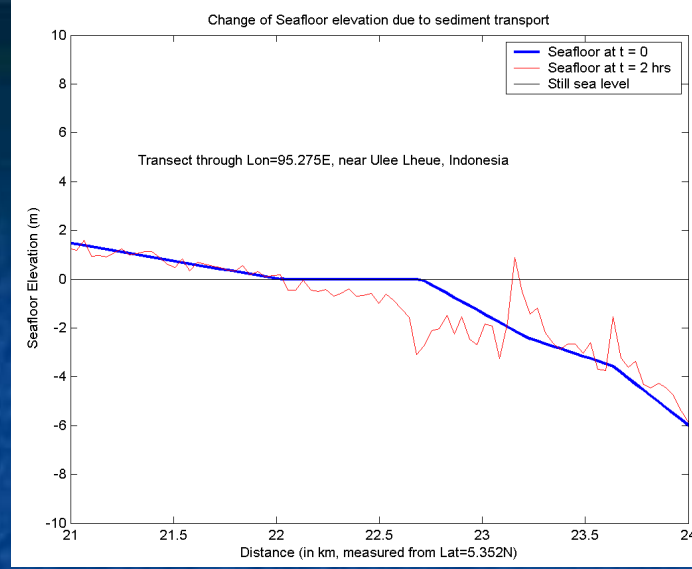
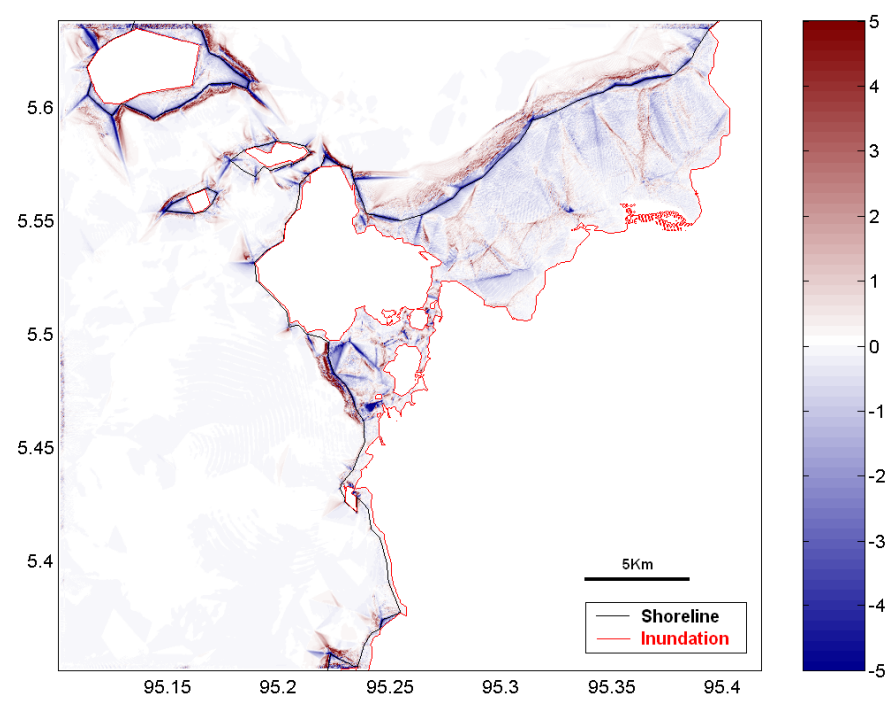
porosity of sand ( $\lambda = 0.3$ )

$$\frac{\mathbf{r}}{q} = 8 \sqrt{\frac{\rho_s - \rho}{\rho} g d_s} (\theta - \theta_c)^{3/2} \frac{\mathbf{v}}{|\mathbf{v}|} \frac{u}{|u|}$$

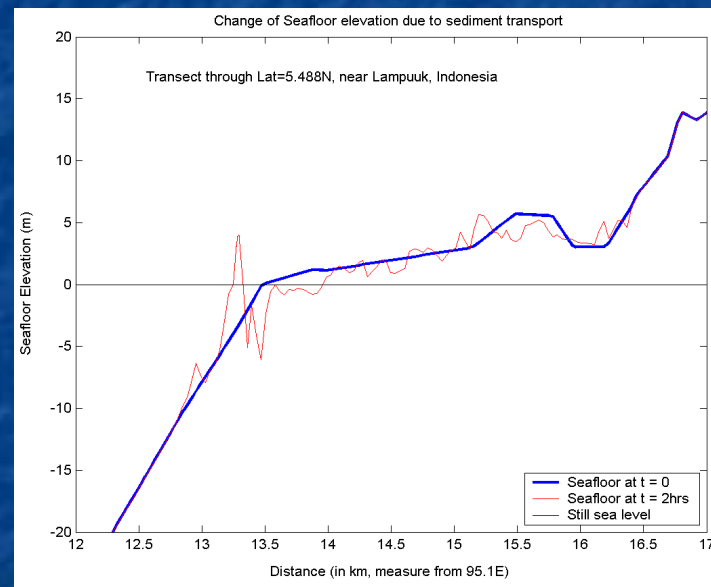
$$\theta_c = 0.06$$

mean sediment diameter  
( $d_s = 0.5\text{mm}$ )

# Ulee Lheue



# Lampuuk



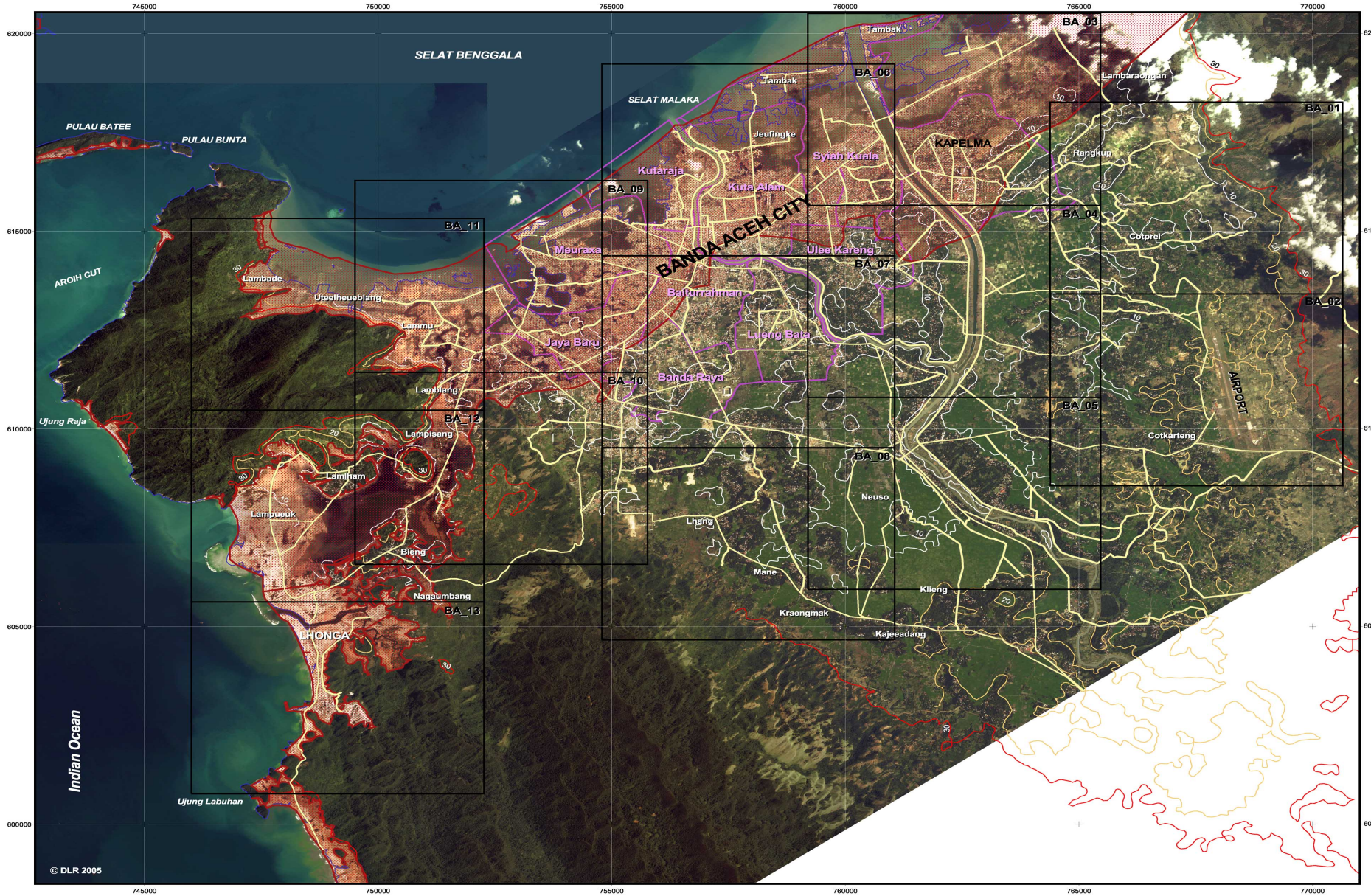
Transect of seafloor elevation. The thick blue line shows the original sea bottom and the red thin line denotes the calculated sea bottom 2 hours after the earthquake. Black line stands for the still sea level.



# Flooding and erosion at Banda Aceh

INDONESIA/SUMATRA - Banda Aceh Region

1 : 45.000





# Concluding Remarks

- **COMCOT provides reasonable results for arrival time, wave height and runup;**
- **Shield parameter is used as an index for potential sediment movement;**
- **A simple sediment transport model is implemented, using the COMCOT results as an driving force.**
- **The sediment transport model needs to be improved and validated.**